

# Problem Set 1

BPHD8110-001

Due: January 26<sup>th</sup>, 2023

1. Show that any strictly dominant strategy in game  $[I, \{\Delta(S_i), \{u_i(\cdot)\}\}]$  must be a pure strategy.
2. Show that the two-player game below has a unique equilibrium:

		Player 2		
		<i>L</i>	<i>C</i>	<i>R</i>
Player 1	<i>U</i>	1, -2	-2, 1	0, 0
	<i>M</i>	-2, 1	1, -2	0, 0
	<i>D</i>	0, 0	0, 0	1, 1

3. This question will test your understanding of Proposition 8.D.1 in MWG. Find all pure and mixed strategy Nash equilibria to the following game. If there are none of either type explain why there are none:

		Player 2				
		F	G	H	I	J
Player 1	A	18, 11	9, 12	6, 1	8, 0	7, 1
	B	6, 6	7, 8	5, 7	9, 11	4, 5
	C	9, 0	4, 5	14, 4	4, 10	5, 16
	D	3, 4	3, 6	2, 3	6, 7	1, 9
	E	0, 0	4, 2	7, 1	7, 4	8, 6

4. Consider the following two-player sequential game. In period 1 each player is given \$1. Player 1 has the opportunity to end the game right away by choosing Down and both players will get \$1. However, Player 1 also have the option of sending the game to a second period by choosing Across, where \$1 will be taken from Player 1 and \$2 will be given to the Player 2. Player 2 then has the opportunity to end the game (with Player 1 getting nothing and Player 2 getting \$3) by choosing Down or to send the game to a third period by choosing Across where \$1 will be taken from him and \$2 dollars will be given to Player 1. Player 1 can then end the game by choosing Down or continue the process by choosing Across. If the game gets to period 8 and Player 2 decides to send the game onto the next period by choosing Across, the game ends with both players getting \$5.
  - a Draw the extensive form version (game tree) of this game. Be sure to include all the components of the game in your diagram.
  - b Find the subgame perfect Nash equilibrium to this game.
  - c What is the outcome if the subgame perfect Nash equilibrium is played?

5. Two investors have each deposited  $D$  with a bank. The bank has invested these deposits in a long-term project. If the bank is forced to liquidate its investment before the project matures, a total of  $2r$  can be recovered, where  $D > r > \frac{D}{2}$ . If the bank allows the investment to reach maturity, however, the project will pay out a total of  $2R$ , with  $R > D$ .

There are 2 dates at which the investors can make withdrawals from the bank: date 1 is before the bank's investment matures; date 2 is after. For simplicity, assume that there is no discounting. If both investors make withdrawals at date 1 then each receives  $r$  and the game ends. If only one investor makes a withdrawal at date 1 then that investor receives  $D$ , the other receives  $2r - D$ , and the game ends. Finally, if neither investor makes a withdrawal at date 1 then the project matures and both investors make withdrawal decisions at date 2. If both investors make withdrawals at date 2 then each receives  $R$  and the game ends. If only one investor makes a withdrawal at date 2 then that investor receives  $2R - D$ , the other receives  $D$ , and the game ends. If neither investor makes a withdrawal at date 2 then the bank returns  $R$  to each investor and the game ends. Note that neither player observes the withdrawal decision of the other player at either date (in other words, at date 1 the players simultaneously choose to withdraw or not, and the same at date 2 – obviously once date 2 is reached both players know what the other player chose at date 1).

- a** Draw the extensive form version of this game.
- b** There are 2 subgames in this game, one of which is the entire game and the other of which is the game that begins at date 2. Write down the normal form version of the subgame that begins at date 2.
- c** Find the Nash equilibrium to the date 2 subgame in part **b**.
- d** Find the subgame perfect Nash equilibria to this game.
- e** Write out the strategic (normal) form of the entire game and find all pure strategy NE.