Problem Set 2

BPHD8110-001

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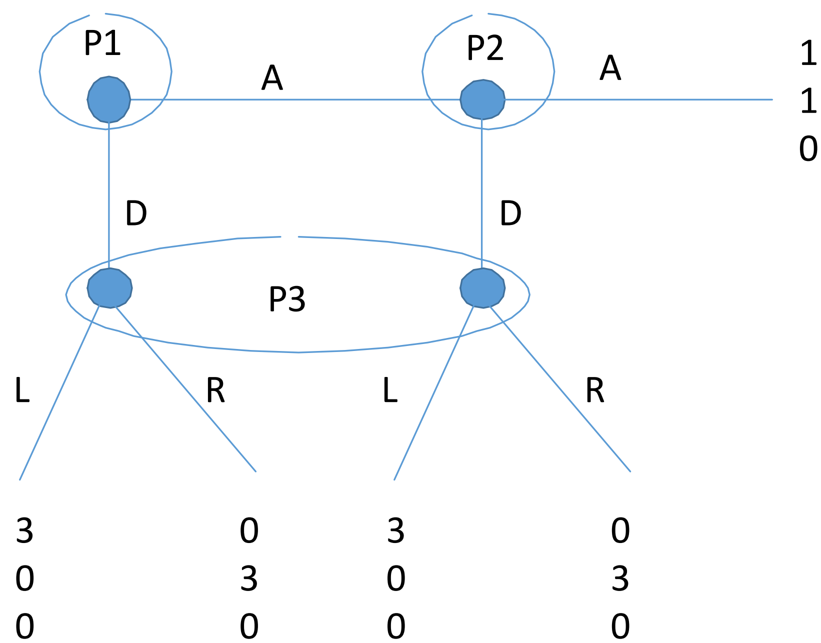
1. Consider the three-player game below, which starts with P1:



a Show that the outcome path (A, A) that results in payoffs $(1, 1, 0)$ is NOT the outcome from a Nash equilbrium.

b Find all pure strategy Nash equilibrium to this game.

2. Consider a developer who wishes to purchase k parcels of land. If the developer purchases all k parcels, the developer receives a payment of D. If the developer does not purchase all k parcels, the developer receives a payment of 0. The developer must purchase each parcel of land from the landowner who owns the land.

   Consider k landowners who each own a parcel of land. That parcel has value of v_i to the landowner, where $v_i \sim U[0, \frac{D}{k}]$. The individual landowners know their own value for the land but the developer does not. Also, the landowners do NOT know the values of other landowners.

   The game can be modeled as a sequential game. The developer makes an offer w_i to each landowner. Each landowner only observes his own w_i and must make a decision to accept or reject that w_i. If all k landowners accept their own offer w_i, then the landowners each receive w_i as a payment from the developer; the developer pays an amount $\sum_{i=1}^{k} w_i$, and the developer receives a payment of D. If ANY landowner chooses to reject w_i, then the developer makes no payment to any landowner and acquires no parcels of land – the developer receives 0 but pays 0. The landowners, who still own their land, receive v_i.

   For simplicity, assume the seller sets $w_i = w_j$ for all i, j. The developer maximizes expected utility, and receives $D - kw$ if aggregation is successful (which occurs only if all k landowners accept the offer) and 0 if not. Note that $\Pr(\widetilde{v} > v)$ for the uniform distribution $U\left[0, \frac{D}{k}\right]$ is $\frac{\widetilde{v}}{\frac{D}{k}}$.

   Assume the developer is risk neutral. Find a subgame perfect Nash equilibrium to this game with k landowners. Be sure to set up the developer's expected utility function correctly.

3. Consider 2 firms who play a simultaneous Cournot game. Market demand is given by $a - bq_1 - bq_2$, with $a > 0$ and $b > 0$. Firm 1 has constant marginal cost of c_1 and Firm 2 has constant marginal cost of c_2 with $c_1 < c_2$ and no fixed costs for either firm. The profit to firm i is:

$$\Pi_i(q_i, q_j) = (a - bq_i - bq_j)\, q_i - c_i q_i$$

   These firms, however, are not solely concerned with profit, but are also concerned with inequity in production. Thus, instead of maximizing profit they maximize *utility* by choosing quantity, where utility is given by:

$$U_i(q_i, q_j) = \Pi_i(q_i, q_j) - \beta\,(q_i - q_j)^2$$

   where $\beta > 0$ is a constant which measures how much the firm dislikes inequity in production.

   a Find the best response functions for this simultaneous Cournot game.

   b Find the pure strategy Nash equilibrium for these firms which dislike inequity.

   c In the standard asymmetric cost Cournot model without inequity the PSNE is $q_i = \frac{a-2c_i+c_j}{3b}$ for $i = 1, 2$. Let q_1^* be Firm 1's equilibrium production in the standard model and $\widetilde{q}_1^*$ be Firm 1's equilibrium production in the model with inequity. Show that if $q_1^* > \widetilde{q}_1^*$ then c_1 must be less than c_2.

4. Consider a capacity-constrained duopoly pricing game. Firm j's capacity is q_j for $j = 1, 2$, and each firm has the same constant cost per unit of output of $c \geq 0$ up to this capacity limit. Assume that the market demand function $x(p)$ is continuous and strictly decreasing at all p such that $x(p) > 0$ and that there exists a price $\widetilde{p}$ such that $x(\widetilde{p}) = q_1 + q_2$. Suppose also that $x(p)$ is concave. Let $p(\cdot) = x^{-1}(\cdot)$ denote the inverse demand function.

   Given a pair of prices charged, sales are determined as follows: consumers try to buy at the low-priced firm first. If demand exceeds this firm's capacity, consumers are served in order of their valuations, starting with high-valuation consumers. If prices are the same, demand is split evenly unless one firm's demand exceeds its capacity, in which case the extra demand spills over to the other firm. Formally, the firms' sales are given by the functions $x_1(p_1, p_2)$ and $x_2(p_1, p_2)$ satisfying:

$$\text{If } p_j > p_i: \qquad \begin{cases} x_i(p_1, p_2) = Min\{q_i, x(p_i)\} \\ x_j(p_1, p_2) = Min\{q_j, Max\{x(p_j) - q_i, 0\}\} \end{cases}$$

$$\text{If } p_2 = p_1 = p: \qquad \left\{ x_i(p_1, p_2) = Min\left\{ q_i, Max\left\{ \frac{x(p)}{2}, x(p) - q_j \right\} \right\} \right.$$

   a Suppose that $q_1 < b_c(q_2)$ and $q_2 < b_c(q_1)$, where $b_c(\cdot)$ is the best-response function for a firm with constant marginal costs of c. Show that $p_1^* = p_2^* = p(q_1 + q_2)$ is a Nash Equilibrium of this game.

   b Argue that if either $q_1 > b_c(q_2)$ or $q_2 > b_c(q_1)$, then no PSNE exists.

5. Consider the following simultaneous game:

Player 2

		w	x	y	z
Player 1	w	$4, 4$	$5, 3$	$6, 2$	$1, 1$
	x	$3, 5$	$6, 6$	$7, 10$	$2, 7$
	y	$2, 6$	$10, 7$	$8, 8$	$2, 11$
	z	$1, 1$	$7, 2$	$11, 2$	$3, 3$

   a Find all pure strategy Nash equilibria to the stage game.

   For parts **b-d** assume the game is repeated infinitely.

   b Find a strategy profile that results in an outcome path in which both players choose x in every period and the strategy profile you have found is a subgame perfect Nash equilibrium (SPNE).

   c Find a strategy profile that results in an outcome path in which both players choose x in every odd period and y in every even period and the strategy profile you have found is a subgame perfect Nash equilibrium.

   d Assume that $\delta = 0.4$, where δ is the discount factor. Find a strategy profile that results in an outcome path in which both players choose y in every period and the strategy profile you have found is a subgame perfect Nash equilibrium.