Problem Set 3

BPHD8110-001

Due: March 23, 2023

1. A professional card player is considering playing a game of cards with an unknown player. The unknown card player may be one of two types, a shark (good player) or a fish (bad player). The probability that the unknown player is a shark is $\frac{4}{5}$, while the probability the unknown player is a fish is $\frac{1}{5}$. Both players simultaneously decide whether or not they will play a game of cards with each other. The payoffs are as follows:

   Stranger (shark)

Professional		Play	Not play
	Play	10,10	12,6
	Not Play	15,12	5,4

   Stranger (fish)

Professional		Play	Not Play
	Play	20,2	10,4
	Not Play	1,12	3,10

   Find all pure strategy Bayes-Nash equilibria to this game.

2. Two partners must dissolve their partnership. Partner 1 currently owns share s of the partnership, partner 2 owns share $1-s$. The partners agree to play the following game: partner 1 names a price, p, for the whole partnership, and partner 2 then chooses either to buy 1's share for ps or to sell her share to 1 for $p(1-s)$. Suppose it is common knowledge that the partners' valuations for owning the whole partnership are independently and uniformly distributed on $[0,1]$, but that each partner's valuation is private information. What is the perfect Bayesian equilibrium?

3. (First-come, first-serve) Suppose that I symmetric individuals wish to acquire the single remaining ticket to a concert. The ticket office opens at 9 a.m. on Monday. Each individual must decide what time to go to get in line: the first individual to get in line will get the ticket. An individual who waits t hours incurs a (monetary equivalent) disutility of βt. Suppose also that an individual showing up after the first individual can go home immediately and so incurs no waiting cost (there are also no travel costs, so an individual who is not first in line incurs no costs at all). Individual i's value of receiving the ticket is θ_i, and each individual's θ_i is independently drawn from a uniform distribution on $[0,1]$.

   a What is the expected value of the number of hours that the first individual in line will wait?

   b How does this vary when β doubles?

   c How does this vary when I doubles?

4. Consider a Cournot game of incomplete information. There are 2 firms in this market. Firms face the following inverse demand function, $P(Q) = 194 - Q$, where $Q = q_1 + q_2$. Firms 1 and 2 may have high or low cost and while each firm knows its own cost the other firm only knows the distribution of costs for its competitor. With probability α Firm 1 has total cost $TC_{1L} = 16q_{1L}$, where q_{1L} is the amount Firm 1 produces when it has low cost, and with probability $(1-\alpha)$ Firm 1 has total cost $TC_{1H} = 32q_{1H}$, , where q_{1H} is the amount Firm 1 produces when it has high cost. With probability θ Firm 2 has total cost $TC_{2L} = 24q_{2L}$, where q_{2L} is the amount Firm 2 produces when it has low cost and with probability $(1-\theta)$ Firm 2 has total cost $TC_{2H} = 40q_{2H}$, where q_{2H} is the amount Firm 2 produces when it has high cost. Let $\alpha = \frac{3}{4}$ and $\theta = \frac{1}{2}$. Both firms simultaneously choose a quantity of production in this market. Find a pure-strategy Bayes-Nash equilibrium to this game.

5. Consider a simultaneous Bertrand pricing game with two firms, I and J. Each firm's demand, conditional on their price relative to the other firm, is given as follows:

if	q_i	q_j
$p_i, p_j > r$	0	0
$p_i > r \geq p_j$	0	M
$r \geq p_i > p_j$	0	M
$r \geq p_i = p_j$	$\frac{M}{2}$	$\frac{M}{2}$

This firm demand function tells us that if one firm prices below the other firm and below some level $r > 0$, then the firm with the lower price captures the entire market and sells the quantity $M > 0$. If the firms both choose the same price and it is less than or equal to r then the firms each sell one half of the market ($\frac{M}{2}$). Note that r and M are fixed amounts that are related in the following way – if the market price is less than or equal to r, then consumers will purchase M units of the good. If the market price is greater than r, then the consumers will purchase 0 units. Note that the price space is continuous, so $p_i, p_j \in [0, \infty)$. Figure 1 shows the demand function and the relationship between r, c_H, and c_L, with the thick solid line representing market demand.

a Suppose that both firms have constant average and marginal cost equal to $c > 0$. Also assume all firms that there is complete information about this cost. What is the pure strategy Nash equilibrium of this game?

b Now consider the case when Firm I has cost c_L and Firm J has cost c_H, with $c_L < c_H < r$. Again assume complete information about these costs. What is the pure strategy Nash equilibrium of this game?

c Now consider the case where the cost for each firm may take one of two values $\{c_L, c_H\}$ where $r > c_H > c_L$ (so there is incomplete information about cost for both firms). Cost c_L occurs with probability ρ and cost c_H occurs with probability $1 - \rho$. The equilibrium to this game involves one type playing a pure strategy and the other type playing a mixed strategy.

- Which type plays a pure strategy and what is that pure strategy?
- The type that plays a mixed strategy chooses its price randomly from a uniform distribution over a particular interval. What is that interval?

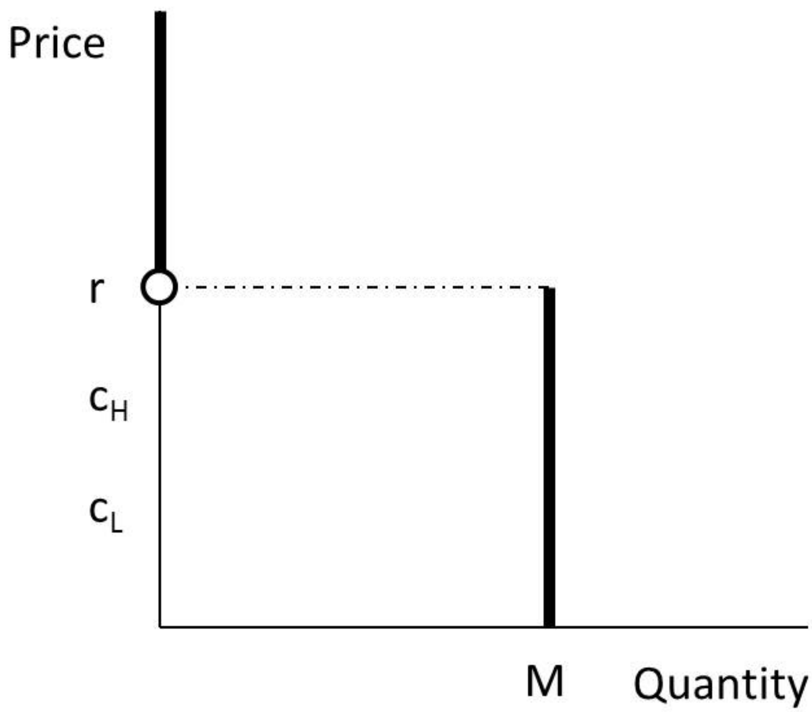

Figure 1: Demand function for Bertrand game