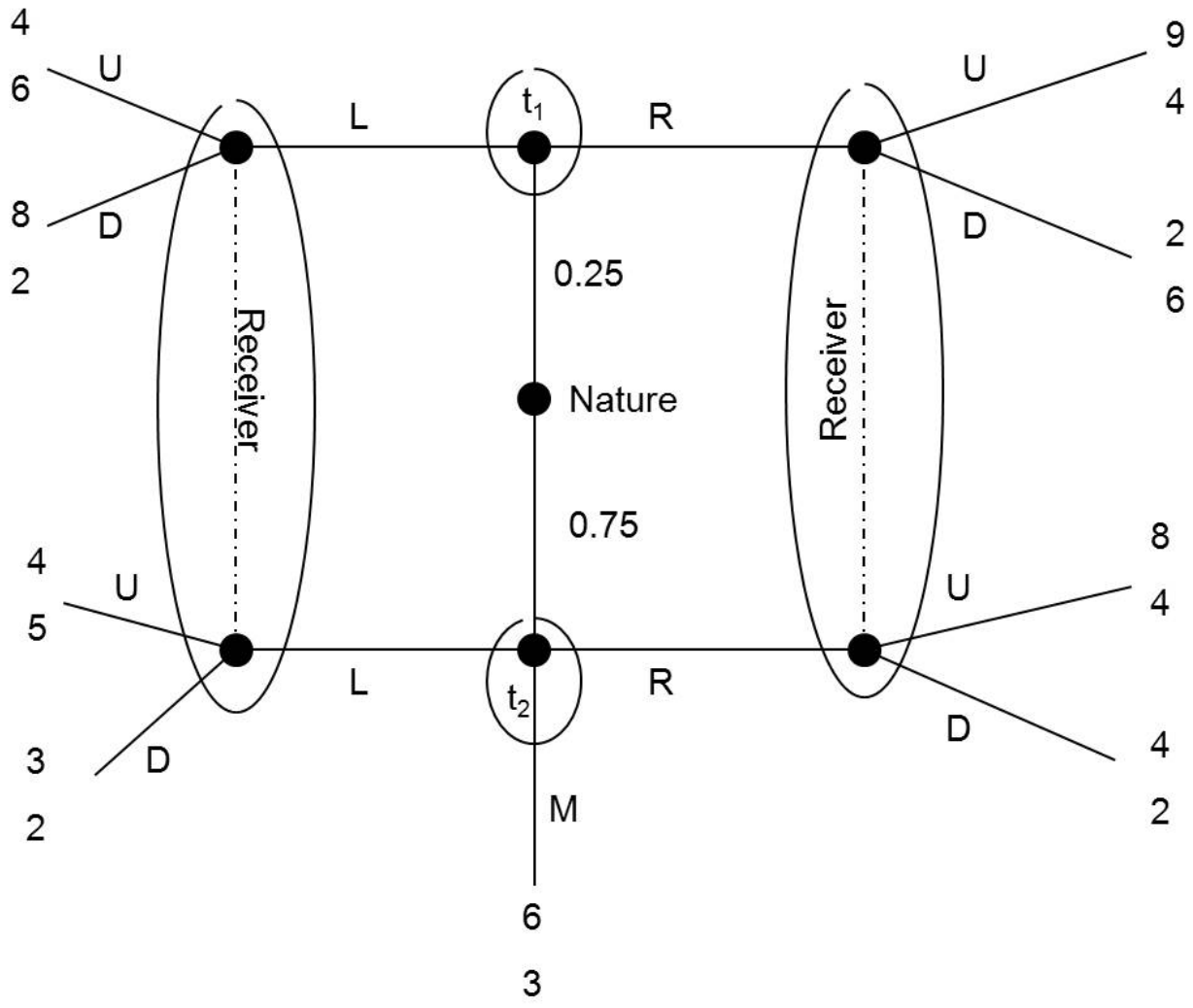
Problem Set 4

BPHD8110-001

Due: April 13

1. Consider the following Sender-Receiver game that has been slightly modified as type t_2 now has a third option, M, which ends the game without allowing the Receiver a chance to make a decision. Note that the probability of being a sender type t_1 is $\frac{1}{4}$ and the probability of being a sender type t_2 is $\frac{3}{4}$.



a Find all pure strategy pooling perfect Bayesian equilibria.

b Find all pure strategy separating perfect Bayesian equilibria.

2. An individual named B.B. invented a device for monitoring the effort level of employees. This device takes precise measurements of the effort of employees and the Dept. of Justice has certified that its measurements are admissible and valid in court proceedings. B.B. now has a problem: how does he price his wonderful new invention?

   Assume that he is trying to sell lit to one particular person named Xavier (X). X has risk neutral preferences and wishes to contract with an Agent, A, to have A sell some books for him. A is risk averse, has utility function of $\text{mu}(w, e) = \sqrt{w} - e$ and can choose either $e_h > e_l$, or not work at all and receive reservation utility of $\overline{u}$. There are 3 possible outcomes for A's efforts. She can either sell many books, a few books, or no books (x_m, x_f, x_0). If e_h is chosen then these states occur with probabilities $(0.75, 0.20, 0.05)$. If e_l is chosen then these states occur with probabilities $(0.20, 0.30, 0.50)$.

   Let $x_m = 600$, $x_f = 200$, $x_0 = 0$, $e_h = 5$, $e_l = 0$, and $\overline{u} = 15$.

   a What contract would X offer to A if X could monitor A perfectly? In other words, what wage would X offer when observing e_h and when observing e_l? Also note the profit X receives in this case.

   b In the imperfect information case, X must offer a wage based on the outcomes observed (x_m, x_f, x_0). Thus X's problem, in order to achieve high effort from the employee, is:

   $$\min (0.75w_m + 0.2w_f + 0.05w_m)$$

   What are the incentive compatibility and participation constraints needed for this problem?

   c Because B.B. is very good at solving optimization problems, once you have set it up he tells you that the solution to this one is $w_m = 433.22$, $w_f = 378.11$, $w_0 = 100.14$. Verify that he is right.

   d Now B.B. knows everything he needs to know in order to set his price for his invention. What is the maximum amount B.B can charge X for the use of his wonderful new device? Explain.

3. Consider a game between a firm and a consumer. The firm may be one of two types, high cost (t_h) or low cost (t_l), where the cost of the firm switching to "green" (or environmentally friendly) production. The firm knows its type and can choose to either switch to using green production or not. The consumer does not observe the firm's type, only the production decision made by the firm. The consumer initially believes that with probability q the firm is a high cost type and with probability $1 - q$ the firm is a low cost type, where $0 \leq q \leq 1$. The consumer's decision is whether or not to buy the product. The consumer has a value of V_P for the product, regardless of whether or not the firm uses green technology. If the firm uses green technology, then the consumer receives additional utility V_E due to the fact that the consumer gets some utility from having purchased from a green producer (so the consumer's total utility is $V_P + V_E$ in this instance). If the consumer chooses not to buy then the consumer receives a value of 0.

   If the firm does not switch to green production then the firm receives a profit of R_{NG} if the consumer purchases (regardless of the firm's type) and 0 if the consumer does not. If the firm makes a claim about using green production then the firm receives a profit of $R_G - c_L$ if the firm is a low cost type and the consumer buys, $R_G - c_H$ if the firm is a high cost type and the consumer buys, $-c_L$ if the firm is a low cost type and does not buy, and $-c_H$ if the firm is a high cost type and the consumer does not buy.

   a Draw a game tree of this dynamic game of incomplete information.

   b Find the restrictions on the parameters such that there is a separating perfect Bayesian equilibrium where the low cost types choose green production and the high cost types do not, while the consumers choose to buy regardless of type.

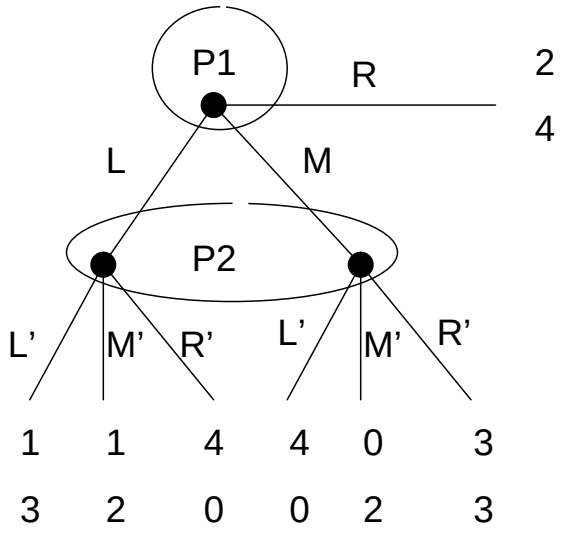
   c Find the restrictions on the parameters such that there is a pooling perfect Bayesian equilibrium where all types choose green production and where the consumers choose to buy regardless of type.

4. Suppose that members of Congress believe that the US legal system needs to be reformed to reduce expenditures on legal costs. Currently both parties pay their own legal costs (call this the Current System). Congress proposes that the loser of the lawsuit pays the winner an amount equal to the loser's costs, so that the loser would have to pay double his costs (call this the Proposed System). The thought is that if the cost to the loser increases, the loser will think twice about going to court because it will cost him more money.

   Let us set up the problem as follows. Assume that each party in the lawsuit has a privately known value of winning the lawsuit relative to losing, and that this value is independently drawn from a common probability distribution over the range $[\underline{v}, \overline{v}]$. Also assume that parties simultaneously and independently decide how much to spend on legal expenses and that whoever spends the most will win the lawsuit, and that the parties are risk-neutral.

   Would we expect the Proposed System to reduce legal expenses relative to the Current System? Clearly explain why, citing specific reasons. **Note**: You should not need a lot of formal mathematics here, nor should you need to use much opinion.

5. Consider the following game:



   a Write down the normal form (or matrix) for this game.

   b Find all pure strategy Nash equilibria (PSNE), subgame perfect Nash equilibria (SPNE), and perfect Bayesian equilibria (PBE) in this game.