Bargaining

Bargaining is one model that can be used to determine how to split surplus between two or more parties. Two approaches are examined, one of which (axiomatic bargaining) can be considered as an arbitrator or mediator attempting to find a rule that will satisfy certain criteria, and the other (alternating offers) which will be analyzed with the standard Nash equilibrium concepts and refinements previously discussed.

1 Axiomatic Bargaining[1]

Nash bargaining is one type of axiomatic bargaining solution that relies on a set of assumptions about the bargaining solution. The structure of the game itself is fairly straightforward. There are I players and a utility set $U \subset \mathbb{R}^I$ that represents all possible allocations of utility for each player. The utility set U is typically assumed to be convex, closed, and satisfies free disposal (if $u' \leq u$, and $u \in \mathbb{R}$, then $u' \in \mathbb{R}$). There is also a disagreement (or threat or status quo) vector $u^* \in U$ that represents the outcome that will occur if an agreement cannot be reached. One type of structure for the game would require each player to submit a utility claim, u_i. If the vector utility claims is feasible, meaning it is available in U, then the players will receive their submitted utility claims. If the set is not feasible, then the players receive their disagreement vector of utilities u^*. One could think about a game where there is a total amount of utility available and if the sum of player claims exceed what is feasible then players receive the disagreement point. However, the axiomatic approach to bargaining is more closely aligned with cooperative game theory (which we have not discussed) than noncooperative game theory (which has been the primary basis for the entire course). Despite being called the Nash bargaining solution, we are not looking for a Nash equilibrium.

Definition 1 *A bargaining solution is a rule that assigns a solution vector $f(U, u^*)$ to every bargaining problem (U, u^*).*

There are a few properties that can be used to impose some structure on the bargaining solution.

Definition 2 *Independence of utility origins: The bargaining solution satisfies this property if, for any $\alpha = (\alpha_1, ..., \alpha_I) \in \mathbb{R}^I$, we have:*

$$\begin{aligned} f_i(U', u^* + \alpha) &= f_i(U, u^*) + \alpha_i \quad \textit{for every } i \textit{ whenever} \\ U' &= \{(u_1 + \alpha_1, ..., u_I + \alpha_I) : u \in U\} \end{aligned}$$

This property means that levels of utility can be rescaled. One benefit of this property is that u^* can be set to zero without loss of generality, so that the disagreement point can be zero.

Definition 3 *Independence of utility units: The bargaining solution satisfies this property if for any $\beta = (\beta_1, ..., \beta_I) \in \mathbb{R}^I$ with $\beta_i > 0$ for all i, we have*

$$\begin{aligned} f_i(U') &= \beta_i f_i(U) \quad \textit{for every } i \textit{ whenever} \\ U' &= \{(\beta_1 u_1, ..., \beta_I u_I) : u \in U\} \end{aligned}$$

When discussing utility functions for consumers, a discussion of the ordinality versus cardinality of the utility function likely occurred. In the standard consumer choice model, utility functions are ordinal so that only the ranking of bundles is important; utility functions can be rescaled so comparisons of the actual

[1]Material is from Chapter 22.E from MWG.

level of utility are typically not meaningful as the utility levels for the same bundle for $\widetilde{u}(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$ and $\widehat{u}(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha} + 1000$ are very different but the utility functions themselves represent the same preferences. Due to the ordinality of utility functions, we do not use them to make interpersonal comparisons of utility. This property of independence of utility units, along with the previous property of independence of utility origins, places a similar requirement on bargaining solutions.

Definition 4 *Pareto property or Paretian: The bargaining solution satisfies this property if, for every U, $f(U)$ is a weak Pareto optimum, that is, there is no $u \in U$ such that $u_i > f_i(U)$ for every i.*

This property means that there are no other feasible vectors such that at least one person could be strictly better off while all others are at the same utility level.

Definition 5 *Symmetry: The bargaining solution satisfies this property if whenever $U \subset \mathbb{R}^I$ is a symmetric set, we have that all the entries of $f(U)$ are equal.*

If the agents are all identical then the gains from cooperating are split equally among the agents.

Definition 6 *Individual rationality: The bargaining solution satisfies this property if $f(U) > 0$.*

This property means that no player can be worse off than the disagreement point. Logically, if the solution made a player worse off than the disagreement then the player could just choose not to be part of the game.

Definition 7 *Independence of irrelevant alternatives: The bargaining solution satisfies this property if, whenever $U' \subset U$ and $f(U) \in U'$, it follows that $f(U') = f(U)$.*

This property means that adding irrelevant alternatives does not change the bargaining solution. If a set of utilities is a subset of some larger set of utilities, and that set of utilities is the solution for the larger set of utilities, then that set of utilities must be the solution when the subset itself is the entire set. If you have seen a discussion of social welfare functions, this property is one that we would like a social welfare function to have because we would like consistency in the solution when irrelevant options are added.

There are a few standard methods to generate a bargaining solution: egalitarian, utilitarian, Nash, and Kalai-Smorodinsky. All these solutions satisfy the independence of utility origins, Paretian, symmetry, and individual rationality properties. The reason the Nash solution is typically used in economics and finance research is because it is the only bargaining solution, as proved in Nash (1950), that satisfies all six properties. The egalitarian solution divides the gains from trade equally among the players. However, in order for the gains from trade to be "equal" the independence of utility units property cannot be satisfied because the solution relies on interpersonal comparisons of utilities. The utilitarian solution, $f_u(U)$, maximizes $\sum_i u_i$ on $U \cap \mathbb{R}^I_+$. As with the egalitarian solution, the utilitarian solution also violates the independence of utility units property because utilities are added so the scale matters. The Kalai-Smorodinsky solution considers the maximum utility that any single player could achieve. Let $u^i(U) \in \mathbb{R}$ be the maximum utility value that player i could achieve by some vector $U \cap \mathbb{R}^I_+$. The thought behind this bargaining solution is to consider that one player has all the bargaining power and, if able to make a take it or leave it offer[2] that does not depend upon the response of the other players, would choose to take the maximum possibility utility value for themselves and leave the other players with zero. Assume some arbitrator or social planner observes these comparisons and makes a Pareto optimal allocation that is proportional to the expected utilities that would occur if the social planner chose, with equal probability, among agents to make the take it or leave it offer. While the Kalai-Smorodinsky solution satisfies the independence of utility units property, it does not satisfy the independence of irrelevant alternatives property.

The Nash solution, $f_n(U)$, maximizes the product of utilities, not the sum of utilities. Alternatively, maximizing the product of utilities is equivalent to maximizing the sum of the natural log of the utilities, $\sum_i \ln u_i$. This solution, as well as the egalitarian and utilitarian solutions, satisfies the independence of irrelevant alternatives property because it is the maximum of a strictly concave function. Unlike those two

[2]In the next section of notes there is discussion of sequential bargaining with take it or leave it offers. The axiomatic bargaining approach differs because the responders in this game have no decision to make – the proposer simply dictates what the solution will be and the others must accept it. This type of game is commonly called a dictator game.

solutions, the Nash solution also satisfies independence of utility units. Consider the following, assuming $\sum_i \ln u_i \geq \sum_i \ln u_i'$:

$$\begin{aligned}
\sum_i \ln \beta_i u_i &= \sum_i \ln \beta_i + \sum_i \ln u_i \\
\sum_i \ln \beta_i + \sum_i \ln u_i &\geq \sum_i \ln \beta_i + \sum_i \ln u_i' \\
\sum_i \ln \beta_i + \sum_i \ln u_i' &= \sum_i \ln \beta_i u_i'
\end{aligned}$$

If $\sum_i \ln u_i \geq \sum_i \ln u_i'$ then we know that $\sum_i \ln \beta_i u_i \geq \sum_i \ln \beta_i u_i'$, which means that the utility values can be scaled up or down and the solution will still be preserved.

2 Alternating Offers[3]

An alternative to the Nash bargaining game is a two player sequential game of alternating offers. Games of this type are commonly called ultimatum games because the final offer is a take it or leave it offer (ultimatum). Unlike the Nash bargaining solution, these games are noncooperative games, so the Nash equilibrium solution concept, and any refinements appropriate for the game, is used to determine the equilibrium behavior.

2.1 Finite horizon

Consider a two player bargaining game with a fixed amount, V, over which players bargain.[4] The players alternate offers; that is, if Player 1 makes the first offer and an agreement is reached then the game ends with each player receiving the proposed offer, but if Player 2 rejects the offer then the game continues to a second round of bargaining in which Player 2 now makes the first offer. However, both players have a discount rate of $\delta \in (0,1)$, and due to the delay in bargaining the amount available in period t is discounted by δ^{t-1}. Assume that the offer space is continuous so that offers can be any real number. The number of periods is finite, T; if the players do not reach an agreement by T then the game ends and both players receive zero. As this game is one of complete and perfect information, the subgame perfect Nash equilibrium concept can be used to determine a set of equilibrium strategies and the outcome that results from those strategies.

2.1.1 Odd number of bargaining rounds

When there are an odd number of bargaining rounds, Player 1 makes both the first and last offer. In the last round Player 2 will accept any offer.[5] Player 1 will offer to keep $\delta^{T-1}V$ and give Player 2 zero. Using that information, we can move to period $T-1$ when Player 2 is making the offer. Both Player 1 and Player 2 know that, if an offer is rejected that Player 1 will propose $\left(\delta^{T-1}V, 0\right)$ as the offer. Player 2 would then make an offer of (Player 1's payoff listed first and Player 2's second) $\left(\delta^{T-1}V, \delta^{T-2}V - \delta^{T-1}V\right)$. This logic can be used to work backwards through the game to determine the optimal offers that should be made at each stage of the game. When the number of bargaining rounds is odd, the offer made by Player 1 in the first stage of the game, which is then accepted by Player 2, leads to the following payoffs for Player 1 and Player 2:

$$\begin{aligned}
\Pi_1^*(T) &= V\left(1 - \delta + \delta^2 - ... + \delta^{T-1}\right) \\
\Pi_2^*(T) &= V - \Pi_1^*(T)
\end{aligned}$$

[3] Material is from Chapter 9, Appendix A of MWG.

[4] You may or may not have seen a similar game earlier in the course on a homework or exam.

[5] Player 2 might reject an offer of \$0 because that is also the amount that will be received if no agreement is reached, so Player 2 is indifferent between accepting an offer of \$0 and rejecting and receiving \$0. With the strategy space being continuous Player 1 can never find a "small enough" amount that is slightly larger than \$0. If the strategy space were discrete we could consider an offer that was the smallest possible increment.

2.1.2 Even number of bargaining rounds

When there are an even number of bargaining rounds, Player 1 makes the first offer and Player 2 makes the last offer. In this case, Player 2 would propose $\left(0, \delta^{T-1}V\right)$ in period T and Player 1 would accept. Player 1 would then propose $\left(\delta^{T-2}V - \delta^{T-1}V, \delta^{T-1}V\right)$ in period $T-1$. Note that the form of payoff is very similar to the odd number of bargaining rounds game only shifted to the other player. If we let $\Pi_1^*(T-1)$ be the payoff to Player 1 in a game with an odd number of periods, that would be the same starting point as Player 2 when the number of periods is even, only discounted by δ. So Player 2 would be able to earn $\delta\Pi_1^*(T-1)$ and Player 1 would earn $V - \delta\Pi_1^*(T-1)$.

Rewriting $\Pi_1^*(T)$ and then considering $T \to \infty$, the payoff becomes:

$$
\begin{aligned}
\Pi_1^*(T) &= V\left(1 + \delta^2 + \delta^4 + ... + \delta^{T-1} - \left(\delta + \delta^3 + ... + \delta^{T-2}\right)\right) \\
\Pi_1^*(T) &= V\left(\frac{1}{1-\delta^2} - \delta\frac{1}{1-\delta^2}\right) \\
\Pi_1^*(T) &= V\left(\frac{1-\delta}{1-\delta^2}\right) \\
\Pi_1^*(T) &= \frac{V}{1+\delta} \\
\Pi_2^*(T) &= \frac{\delta V}{1+\delta}
\end{aligned}
$$

2.2 Infinite horizon

While there may be a firm deadline in which a bargain must be made, it is also possible that the players have created a self-imposed deadline of T periods. However, if the players reach period T and have not reached an agreement, they may find it beneficial to continue bargaining. As the number of periods T is then not known, or alternatively the endpoint is uncertain, this game can be modeled as an infinitely repeated game. There is a unique SPNE of this infinitely repeated game that has, in the first period of the game, Player 1 offering Player 2 exactly the limit of the finite period bargaining game previously discussed. In the first period, Player 1 proposes $\left(\frac{V}{1+\delta}, \frac{\delta V}{1+\delta}\right)$ and Player 2 accepts. While not the complete equilibrium strategies (the strategies at all future points of the game are not specified), the outcome of the infinitely repeated bargaining game is the same as the limit of the outcome in the finitely repeated bargaining game. Note that this game differs from our earlier discussion of infinitely repeated games because in the earlier discussion we assumed the players were playing a simultaneous game that was repeated forever. In the bargaining game, players take turns making offers. It is not always the case that the outcome of the limit of a finite period game is the same as the outcome from an infinitely repeated game.

3 Nash Bargaining in Recent Finance Literature

This section is not meant to be a comprehensive overview of papers that use Nash bargaining but to provide some examples of the types of models in which Nash bargaining has recently been used. A common area in which Nash bargaining is used in some capacity is entrepreneurship and venture capital. Gennaioli and Rossi (2013) model optimal debt structure to resolve financial distress. They use Nash bargaining to extend their main results to a case of limited commitment in which renegotiation is possible. Green and Liu (2021) construct a model in which borrowers obtain loans from multiple creditors. In their baseline model, the borrower makes take it or leave it offers to lenders. They extend the model by using Nash bargaining to consider a case where the borrower does not have all the bargaining power. Hu and Varas (2021) consider how a lender obtains private information from a borrower from the lending relationship, which then creates asymmetric information in the market because the existing lender has more information than potential future lenders. While they assume the lender has all the bargaining power, they show that the entrepreneur's financial constraint limits the size of the repayment that can be made and that the Nash bargaining outcome does not always maximize the joint surplus. Hochberg, Ljungqvist, and Vissing-Jorgensen (2014) consider the case of a venture capitalist whose current investors obtain private information about skill whereas potential investors only obtain information about returns. They use Nash bargaining to determine the follow-on fund fee paid by limited partners (current investors). Cong and Xiao (2022) consider the conventional wisdom that persistent performance in venture capital is evidence of skill, and build a model of delegated investment to show how persistent performance could arise without skill differences. They consider Nash bargaining between a general partner (fund manager) and limited partners.[6] Lee and Parlour (2022) study the implication of crowdfunding by consumers rather than traditional lenders or investors. In their model, the product market price is determined by Nash bargaining.

.Nash bargaining also appears as a model to split surplus according to (typically exogenous) bargaining power. It is used in OTC markets (Bolton, Santos, Scheinkman, 2021; Colliard, Foucault, and Hoffman, 2021; Glebkin, Yueshen, and Shen, 2023) where dealers and clients determine prices by Nash bargaining. Foucault, Kadan, and Kandel (2013) use it as a method of splitting gains from trade in between market makers and takers in electronic markets. Sambalaibat (forthcoming) builds a search model (which is also the approach in Glebkin, Yueshen, and Shen) of bond and credit default swaps and models prices between buyers and sellers as a Nash bargaining process. Bai (2021) builds a search model of equilibrium unemployment and uses Nash bargaining to determine wages between firms and employees. Malenko and Malenko (2015) examine leveraged buyouts and club deals. Nash bargaining is the process by which members of the club split the surplus if a takeover bid is successful. Dessaint et al. (2021) also use Nash bargaining in a takeover setting. Donaldson, Gromb, and Piacentino (2020) construct a model in which collateral helps determine the bargaining strength of creditors in a Nash bargaining game. Craig and Ma (2022) use Nash bargaining to determine the split of surplus between borrowing and intermediary banks. Lehar, Song, and Yuan (2020) examine how trade credit can serve as a collusion mechanism in supply chains and use Nash bargaining to determine prices between a retailer and supplier.

Again, by no means is this list exhaustive nor do these brief descriptions of the papers convey the full results of the papers. This section is simply meant as a very brief discussion of some examples of how Nash bargaining is used in top finance research.

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