BPHD 8110 Answers

Test 2

Thursday April 20^{th}

1. (40 points) Consider a risk-neutral principal who has a job for a risk-averse agent. The agent has utility function $u(w,e) = v(w) - g(e)$. The agent can supply one of two levels of effort, e_H or e_L. There are two possible profit outcomes: $\pi_1 = 20$ $\pi_2 = 4$. The probabilities of the profit outcomes conditional on e_H and e_L are: $f(\pi_1|e_H) = \frac{4}{5}$ and $f(\pi_2|e_H) = \frac{1}{5}$; $f(\pi_1|e_L) = \frac{1}{4}$ and $f(\pi_2|e_L) = \frac{3}{4}$. Let $v(w) = \sqrt{w}$ and $g(e_H) = 2$ and $g(e_L) = 1$. The agent's reservation utility is $\overline{u} = 2$.

   a (5 points) What is the principal's expected profit if the agent uses:

   – high effort?

   Answer:

   Leave the wage as w_H and let Π_H be the expected profit minus the wage.

$$\begin{aligned}
\Pi_H &= \pi_1 f(\pi_1|e_H) + \pi_2 f(\pi_2|e_H) - w_H \\
\Pi_H &= 20 * \frac{4}{5} + 4 * \frac{1}{5} - w_H \geq 0 \\
\Pi_H &= \frac{84}{5} - w_H
\end{aligned}$$

   – low effort?

   Answer:

   Leave the wage as w_L and let Π_L be the expected profit minus the wage.

$$\begin{aligned}
\Pi_L &= \pi_1 f(\pi_1|e_L) + \pi_2 f(\pi_2|e_L) - w_L \\
\Pi_L &= 20 * \frac{1}{4} + 4 * \frac{3}{4} - w_L \\
\Pi_L &= \frac{32}{4} - w_L
\end{aligned}$$

   b (5 points) Suppose that effort is observable, so that the principal can pay the agent conditional on effort. What are the wages (w_H and w_L) offered for e_H and e_L? Be sure to compare to the principal's expected profit to make sure the principal will offer a wage.

   Answer:

   When effort is observable, if the principal offers a wage for an effort level, the principal simply offers $w(e_H)$ and $w(e_L)$ such that the relevant participation constraint is satisfied. The participation constraint for high effort is:

$$\begin{aligned}
u(w(e_H), e_H) &\geq \overline{u} \\
\sqrt{w(e_H)} - g(e_H) &\geq \overline{u} \\
\sqrt{w(e_H)} - 2 &\geq 2 \\
\sqrt{w(e_H)} &\geq 4 \\
w(e_H) &\geq 16
\end{aligned}$$

The participation constraint for low effort is:

$$\begin{aligned} u\left(w\left(e_L\right), e_L\right) &\geq \overline{u} \\ \sqrt{w\left(e_L\right)} - g\left(e_L\right) &\geq \overline{u} \\ \sqrt{w\left(e_L\right)} - 1 &\geq 2 \\ \sqrt{w\left(e_L\right)} &\geq 3 \\ w\left(e_L\right) &\geq 9 \end{aligned}$$

If the principal offers a contract, there is no reason to offer more than needed to meet the agent's reservation wage, so if a contract is offered for the two effort levels it will be $w\left(e_H\right) = 16$ and $w\left(e_L\right) = 9$.

The principal expects to receive 16.8 in revenue if high effort is used, so the principal will pay 16 for high effort. However, the principal only expects to receive 8 if low effort is used, so the principal will not pay 9 to induce low effort.

So the contract is $w\left(e_H\right) = 16$ and $w\left(e_L\right) = 0$.

c (10 points) Suppose that effort is now unobservable, so that now the principal must pay wages (w_1 and w_2) based upon the observed profit outcomes of π_1 and π_2.

- What is the agent's participation constraint that must be satisfied?

Answer:

We know from class that the unobservable case will never be more profitable than the observable case, so we only need to be concerned with a participation constraint for high effort.

$$\begin{aligned} \sqrt{w_{\pi_1}} f\left(\pi_1 | e_H\right) + \sqrt{w_{\pi_2}} f\left(\pi_2 | e_H\right) - g\left(e_H\right) &\geq \overline{u} \\ \sqrt{w_{\pi_1}} \frac{4}{5} + \sqrt{w_{\pi_2}} \frac{1}{5} - 2 &\geq 2 \\ 4\sqrt{w_{\pi_1}} + \sqrt{w_{\pi_2}} &\geq 20 \end{aligned}$$

- What is the agent's incentive compatibility constraint that must be satisfied?

Answer:

We did not have to use the incentive compatibility constraint when effort was observable because the principal could set the wage conditional on effort. However here we need:

$$\begin{aligned} &\sqrt{w_{\pi_1}} f\left(\pi_1 | e_H\right) + \sqrt{w_{\pi_2}} f\left(\pi_2 | e_H\right) - g\left(e_H\right) \geq \sqrt{w_{\pi_1}} f\left(\pi_1 | e_L\right) + \sqrt{w_{\pi_2}} f\left(\pi_2 | e_L\right) - g\left(e_L\right) \\ &\sqrt{w_{\pi_1}} \frac{4}{5} + \sqrt{w_{\pi_2}} \frac{1}{5} - 2 \geq \sqrt{w_{\pi_1}} \frac{1}{4} + \sqrt{w_{\pi_2}} \frac{3}{4} - 1 \\ &16\sqrt{w_{\pi_1}} + 4\sqrt{w_{\pi_2}} - 40 \geq 5\sqrt{w_{\pi_1}} + 15\sqrt{w_{\pi_2}} - 20 \\ &11\sqrt{w_{\pi_1}} - 11\sqrt{w_{\pi_2}} \geq 20 \end{aligned}$$

d (5 points) The principal's optmization problem is:

$$\min_{w_1, w_2} \; w_1 f\left(\pi_1 | e_H\right) + w_2 f\left(\pi_2 | e_H\right)$$

subject to the constraints you found in part **c**. Why will the principal's profit maximization problem and cost minimization problem yield the same solution?

Answer:

The wage contract is the choice variable of the principal. While wages affect whether or not an agent uses high effort or low effort, they do not affect the conditional probabilities $\left\{f\left(\pi_1 | e_H\right), f\left(\pi_2 | e_H\right), f\left(\pi_1 | e_L\right), f\left(\pi_2 | e_L\right)\right\}$ so the principal knows what the expected revenue is if the principal can guarantee the agent will exert a particular effort level. The part that determines the profit is the wage contract, and if revenue is constant then the principal minimizing wages is equivalent to maximizing profit. In part **a** we can see that expected revenue from high effort is constant (it is $\frac{84}{5}$) as is expected revenue from low effort (it is 8). As the wage contract does not affect the expected revenue conditional on effort, the revenue can be pulled out of the profit maximization problem to simply focus on minimizing the wage cost.

e (5 points) Find the set of first-order conditions that you would need to solve this problem. Do not spend time trying to find the numeric solution (that is why we have computers, and the numeric solution is in part **f**, so you will not need to use the FOCs in part **f** but I want to see them).

Answer:

The principal solves:

$$\begin{aligned}\mathcal{L}\left(w_{\pi_1}, w_{\pi_2}, \lambda_1, \lambda_2\right) &= \\ & \min w_{\pi_1}\frac{4}{5} + \frac{1}{5}w_{\pi_2} + \lambda_1\left(20 - 4\sqrt{w_{\pi_1}} - \sqrt{w_{\pi_2}}\right) \\ & +\lambda_2\left(20 - 11\sqrt{w_{\pi_1}} - 11\sqrt{w_{\pi_2}}\right)\end{aligned}$$

The first-order conditions are:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial w_{\pi_1}} &= \frac{4}{5} - \lambda_1\frac{4}{2}\left(w_{\pi_1}\right)^{-1/2} - \lambda_2\frac{11}{2}\left(w_{\pi_1}\right)^{-1/2} = 0 \\ \frac{\partial \mathcal{L}}{\partial w_{\pi_2}} &= \frac{3}{8} - \lambda_1\frac{3}{2}\left(w_{\pi_2}\right)^{-1/2} - \lambda_2\frac{1}{2}\left(w_{\pi_2}\right)^{-1/2} = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda_1} &= 20 - 4\sqrt{w_{\pi_1}} - \sqrt{w_{\pi_2}} = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda_2} &= 20 - 11\sqrt{w_{\pi_1}} - 11\sqrt{w_{\pi_2}} = 0\end{aligned}$$

f (10 points) The equilibrium wages are $w_1 = \frac{2304}{121} \approx 19.04$ and $\frac{784}{121} \approx 6.48$.

- Show that the participation and incentive compatibility constraints are satisfied. Use the approximations because the principal cannot really pay a wage of $\frac{2304}{121}$. Note that the constraints will not hold with equality because the wages are approximations, but they should satisfy the greater than criterion.

Answer:

All we need to show is that:

$$\begin{aligned}4\sqrt{w_{\pi_1}} + \sqrt{w_{\pi_2}} &\geq 20 \\ 4\sqrt{19.04} + \sqrt{6.48} &= 20\end{aligned}$$

Technically this may have been slightly less than 20 because I rounded the $\frac{2304}{121}$ down to 19.04 and should have rounded up to 19.05.

Then we also need to show

$$\begin{aligned}11\sqrt{w_{\pi_1}} - 11\sqrt{w_{\pi_2}} &\geq 20 \\ 11\sqrt{19.04} - 11\sqrt{6.48} &\geq 20\end{aligned}$$

Again, technically this may have been slightly less than 20 because I rounded the $\frac{2304}{121}$ down to 19.04 and should have rounded up to 19.05.

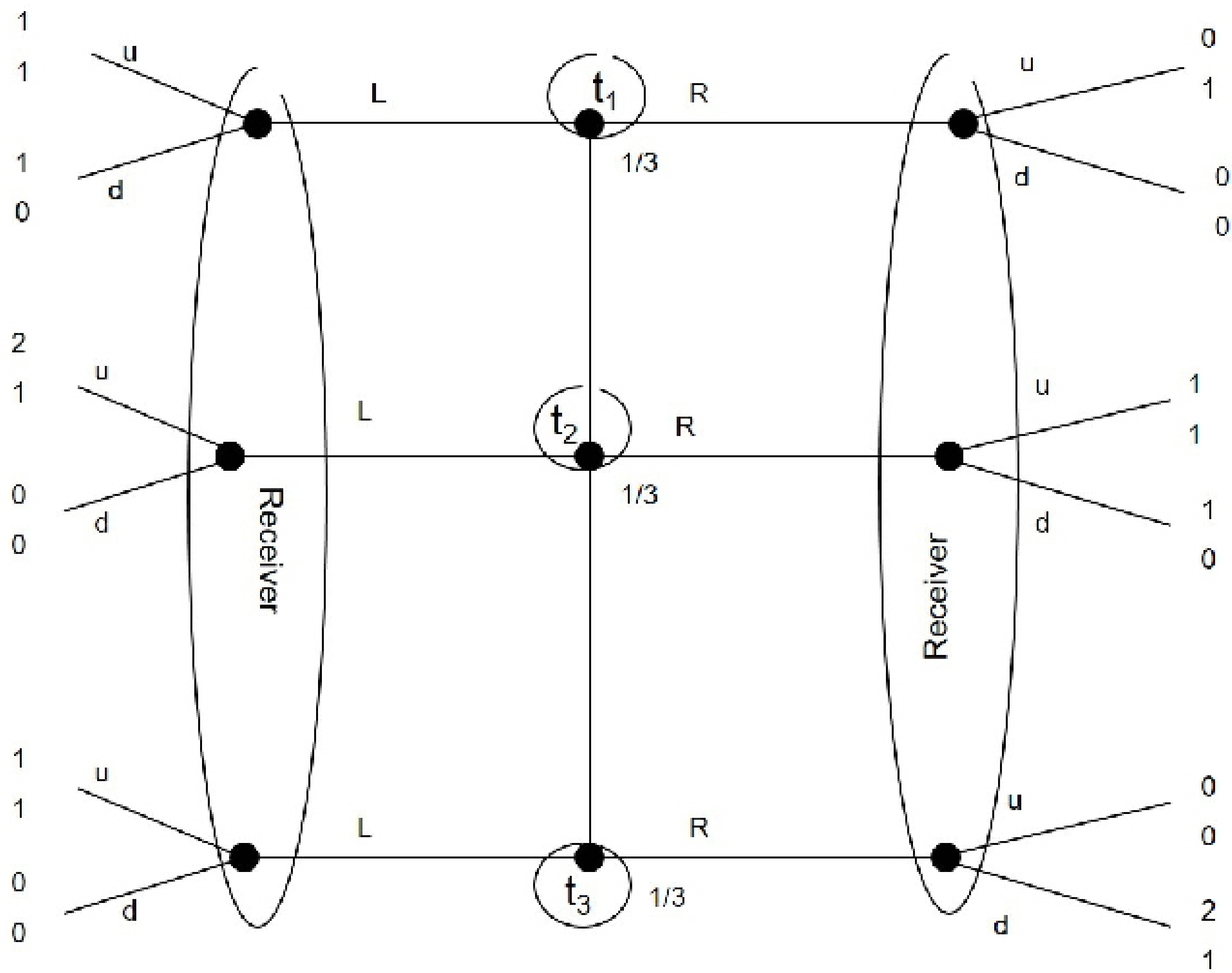
- Show that the principal earns more when effort is observable, even though high effort is exerted in both the observable and unobservable cases.

Answer:

When effort is observable the principal expects to earn $16.8 - 16 = 0.8$.

When effort is unobservable the principal expects to earn $\frac{4}{5}(20 - 19.04) + \frac{1}{5}(4 - 6.48) = 0.272$

2. (30 points) Consider the following dynamic game of incomplete information:



Note that in this game there is a Sender who is one of three types, t_1, t_2, or t_3. Nature determines the Sender's type with equal probability, and the Receiver only observes the Sender's action, R or L, and not the Sender's type. Because there are three types but only two actions the Sender can take, we cannot have a true separating equilibrium where each type perfectly identifies itself by choosing a unique action. However, we can still have pooling equilibria, and we can also have semi-pooling equilibria in which two types choose one action and one type chooses the other action.

a (5 points) There is one type of Sender for whom choosing either Left or Right is a dominant action. Which sender is that and which action will that sender choose?

Answer:

Type t_1 will only ever choose L because if t_1 chooses L then t_1 is guaranteed to get 1 regardless of whether the Receiver chooses Up or Down. If type t_1 chooses R then t_1 is guaranteed to receive 0 regardless of whether the Receiver chooses Up or Down. So t_1 will never choose R.

b (5 points) For one of the Sender actions of Left or Right, the Receiver has a dominant action of either always choosing Up or always choosing Down. What is the dominant action by the Receiver and which action by the Sender, Left or Right, leads to the Receiver using a dominant action?

Answer:

If L is chosen, the Receiver always receives a payoff of 1 if u is chosen, and always receives a payoff of 0 if d is chosen. So if the Receiver observes an action of L then the Receiver will choose u.

Hint: Use the information from parts **a** and **b** to help reduce the set of equilibria you need to check in parts **c** and **d** of this problem

c (10 points) Find all pure strategy pooling perfect Bayesian equilibria to this game. If there are none explain why there are none.

Answer:

We know type t_1 will always choose L (from part **a**) so there will be no pooling equilibrium where all types choose R. The only potential pooling equilibrium involves all Sender types choosing L. We also know that if L is chosen then the Receiver will choose u (from part **b**). There is a third important feature (call this **result c**) of the game that builds upon part **b**. Because the Receiver will always choose u if L, a type t_2 will always choose L. The type t_2 Receiver makes this choice because he will receive 2 from choosing L (because he knows the Receiver will choose u if L) and 1 regardless of whether the Receiver chooses u or d if he chooses R.

So the only need is to focus on what happens off the equilibrium path if R is chosen.

Thus, the Receiver's beliefs will be that if L is observed it is equally likely that it is type t_1, t_2, or t_3. By part **b** we know the Receiver will choose u if L. We know, from **a** and **result c**, that types t_1 and t_2 will not deviate, so we just need to check type t_3. Type t_3 will deviate if the Receiver chooses d if R, so we need to check the probabilities that ensure the Receiver chooses u if R.

Let p be the belief that the Receiver has that a type t_1 chooses R, q be the belief the Receiver has that a type t_2 chooses R, and $1-p-q$ be the belief that the Receiver has that a type t_3 chooses R.

The Receiver's expected value of choosing u if R is:

$$E[u|R] = 1*p + 1*q + 0*(1-p-q) = p+q$$

The Receiver's expected value of choosing d if R is:

$$E[d|R] = 0*p + 0*q + 1*(1-p-q) = 1-p-q$$

Again, we need $E[u|R] \geq E[d|R]$ in order for type t_3 to not switch to R. So:

$$\begin{aligned} p+q &\geq 1-p-q \\ 2p+2q &\geq 1 \\ p+q &\geq \frac{1}{2} \end{aligned}$$

So the Receiver will choose u if R if he believes it is less than 50% likely that a type t_3 is choosing R. That seems plausible given the initial probabilities, and given that those information sets are off-the-equilibrium path any probabilities are permissible in a weak perfect Bayesian equilibria provided they abide by the laws of probability. Thus, we have a WPBE such that:

$$\text{Sender's Strategy} \begin{cases} t_1 \text{ choose } L \\ t_2 \text{ choose } L \\ t_3 \text{ choose } L \end{cases}$$

$$\text{Receiver's Beliefs} \begin{cases} \Pr(t_1|L) = \frac{1}{3} \\ \Pr(t_2|L) = \frac{1}{3} \\ \Pr(t_3|L) = \frac{1}{3} \\ \Pr(t_1|R) = p \\ \Pr(t_2|R) = q \\ \Pr(t_3|R) = 1 - p - q \end{cases}$$

$$\text{Receiver's Strategy} \begin{cases} \text{Choose } u \text{ if } L \\ \text{Choose } u \text{ if } R \end{cases}$$

where $p + q \geq \frac{1}{2}$.

Taking this problem one step further, we should be able to see that this equilibrium is NOT a strong perfect Bayesian equilibrium. The Receiver should know that, given **a**, **b**, and **result c**, neither type t_1 nor type t_2 should be choosing R.

d (10 points) Find all pure strategy semi-pooling perfect Bayesian equilibria to this game. If there are none explain why there are none.

Answer:

From **a**, **b**, and **result c** we know that t_1 will always choose L; the Receiver will always choose u if L; and t_2 would always choose L. The only viable semi-pooling equilibrium is for type t_3 to choose R. If that happens, then the Receiver knows that type t_3 is choosing R with certainty and will choose d. We know types t_1 and t_2 will not switch from L, and type t_3 is now earning 2 from choosing R and would only earn 1 if she switched to L. Thus, we have the following semi-pooling equilibrium:

$$\text{Sender's Strategy} \begin{cases} t_1 \text{ choose } L \\ t_2 \text{ choose } L \\ t_3 \text{ choose } R \end{cases}$$

$$\text{Receiver's Beliefs} \begin{cases} \Pr(t_1|L) = \frac{1}{2} \\ \Pr(t_2|L) = \frac{1}{2} \\ \Pr(t_3|L) = 0 \\ \Pr(t_1|R) = 0 \\ \Pr(t_2|R) = 0 \\ \Pr(t_3|R) = 1 \end{cases}$$

$$\text{Receiver's Strategy} \begin{cases} \text{Choose } u \text{ if } L \\ \text{Choose } d \text{ if } R \end{cases}$$

Note that $\Pr(t_1|L) = \Pr(t_2|L) = \frac{1}{2}$. That is not a typo or assumption – both type t_1 and t_2 occur in nature with probability $\frac{1}{3}$, but the Receiver knows t_3 is choosing R, so $\Pr(t_3|L) = 0$. Thus, the Receiver updates his beliefs about the likelihood of the type being t_1 or t_2 given that L is observed. The probabilities conditional on observing L are only equal because the initial probabilities of t_1 and t_2 are equal. If t_1 had occurred in nature with $\frac{1}{6}$ chance and t_2 with $\frac{3}{6}$ chance, then $\Pr(t_1|L) = \frac{1/6}{4/6} = \frac{1}{4}$ and $\Pr(t_2|L) = \frac{3/6}{4/6} = \frac{3}{4}$.

Note that this semi-pooling equilibrium meets the requirements for a strong perfect Bayesian equilibrium. Of the Bayesian equilibria, I would view the semi-pooling equilibrium as more likely to emerge than the pooling equilibrium. Both players should know that types t_1 and t_2 would never choose R, making it obvious to the Receiver that if R is observed it must be type t_3.

3. (10 points) **Problem set-up**: In an all pay auction, all bidders submit a single sealed bid, the highest bid wins, but all bidders (even the losing bidders) must pay their bid. The lowest valued user expects zero surplus. Assume that the bidders are risk-neutral and that we have the symmetric independent private values environment where $v_i \tilde{} U[\underline{v}, \overline{v}]$. Also assume that the bid functions are strictly monotonic so that individuals who draw higher values will submit higher bids and thus the highest valued bidder will win the item. Consider this mechanism the "All Pay" mechanism.

   Now consider a raffle (or lottery) in which individuals purchase tickets to win a good, and each ticket purchased gives the individual one additional entry into the raffle. A winning ticket is drawn randomly from the purchased tickets and the winner is the individual who purchased that randomly drawn ticket. The raffle is similar to the all pay auction in that all individuals pay but only one individual wins the item. Assume that participants in the raffle make any ticket purchases at the same time (effectively making the raffle a simultaneous game direct mechanism in that all participants purchase tickets at the same time and if any participant purchases multiple tickets that participant purchases those tickets at the same time, like a sealed bid in an auction). Assume that the raffle participants are risk-neutral and that we have the symmetric independent private values environment where $v_i \tilde{} U[\underline{v}, \overline{v}]$ is the value draw of the item for a particular participant i. Assume that bidders with higher value draws will purchase more tickets. Consider this mechanism the "Raffle" mechanism.

   The two mechanisms are very similar in that all participants pay an amount of money, but in the All Pay mechanism the participant who spends the most wins whereas in the Raffle mechanism the participant who spends the most has the highest probability of winning, but is not guaranteed to win.

   Question: Assume that there are the same number of bidders in the All Pay mechanism as there are participants in the Raffle mechanism. Can the Revenue Equivalence Theorem be used to show revenue equivalence between the All Pay and Raffle mechanisms? Explain why or why not with reference to the requirements needed for the Revenue Equivalence Theorem to hold.

   Answer:

   To begin with, many assumptions of the Revenue Equivalence Theorem are met – both mechanisms have players with values drawn from a SIPV environment, players are risk neutral, and there are the same number of players in each mechanism. By assumption, players with higher values bid higher/buy more tickets so that players with higher values should have a higher probability of winning.

   The key is that the Raffle does not guarantee that the player with the highest value wins the item even if the player with the highest value spends the most money. That random allocation rule violates the condition that players with the same value across the two mechanisms have the same probability of winning. Assume (for now) that at least two participants buy raffle tickets and one of those participants has the highest possible value $\overline{v}$. A participant with value $\overline{v}$ will win with certainty in the All Pay mechanism, but will not win with certainty if any other participant chooses to purchase a ticket in the Raffle mechanism. Now suppose that the equilibrium is that only a bidder with value $\overline{v}$ will purchase a ticket in the raffle, so that the potential equilibrium is that one ticket is sold only in the case when a bidder with value $\overline{v}$ is present. If that is the equilibrium it should be possible for some other bidder with a value of $\overline{v} - \varepsilon$ to buy a single ticket to win the item with 50% probability.

   Another way to think about the revenue equivalence between the two mechanisms is to consider a bidder who draws $\underline{v}$. There is zero probability that a bidder who draws the lowest possible value can win in the All Pay mechanism; however, there is a chance that bidder could win in the raffle. You might argue that someone who draws the lowest value may not bid or buy a ticket – the lowest valued player not participating could be part of the equilibrium and there is no guarantee, without working through all the math to find the equilibrium, that the lowest valued bidder will buy a ticket in the raffle. But the condition about the probability of winning being the same across mechanisms would be violated here if we assumed a player with value $\underline{v}$ does buy a ticket because there is always a chance of winning in the Raffle.

   You only have to show that one condition is violated in order to show that the Revenue Equivalence Theorem does not hold. Also, to be clear, it is still possible that the two mechanisms generate the same expected revenue – all we have shown is that we cannot use the Revenue Equivalence Theorem to make that claim.

4. (20 points) Consider the following game of incomplete information between Player 1 and Player 2. Player 1's type is known but Player 2 may be either an H type (with probability p) or an L type (with probability $1-p$). The payoffs to this simultaneous game of incomplete information are as follows:

Player 2 (H type)

Player 1		x	y
	a	$1,3$	$1,2$
	b	$3,1$	$2,5$

Player 2 (L type)

Player 1		x	y
	a	$3,2$	$1,3$
	b	$2,1$	$0,4$

a (10 points) Suppose that $p = 0.75$. Find all pure strategy Bayes-Nash equilibria under this assumption.

Answer:

Suppose that Player 1 chooses a. The H type's best response is to choose x and the L type's best response is to choose y. Player 1's expected value of choosing a is:

$$E\left[a\right] = 1 * \frac{3}{4} + 1 * \frac{1}{4} = 1$$

Player 1's expected value of choosing b is:

$$E\left[b\right] = 3 * \frac{3}{4} + 0 * \frac{1}{4} = \frac{9}{4}$$

Because $E\left[b\right] > E\left[a\right]$, this is is NOT an equilibrium.

Suppose that Player 1 chooses b. The H type's best response is to choose y and the L type's best response is to choose y. Player 1's expected value of choosing b is:

$$E\left[b\right] = 2 * \frac{3}{4} + 0 * \frac{1}{4} = \frac{6}{4}$$

Player 1's expected value of choosing a is:

$$E\left[a\right] = 1 * \frac{3}{4} + 1 * \frac{1}{4} = 1$$

Because $E\left[b\right] > E\left[a\right]$, this is a pure strategy Bayes-Nash equilibrium. Thus, Player 1 choose b, Player 2 (H type) choose y, and Player 2 (L type) choose y is a pure strategy Bayes-Nash equilibrium.

b (10 points) Find all pure strategy Bayes-Nash equilibria for each value of p (because p is a probability $p \in [0,1]$).

Hint 1: There are no values of p such that there is more than one pure strategy Bayes-Nash equilibrium for that value of p.

Hint 2: It is best to find ranges of p for which a specific equilibrium exists.

Hint 3: There is a range of p for which there are no pure strategy Bayes-Nash equilibria.

Answer:

If Player 1 chooses a, Player 2's best response is x if H type and y if L type. Player 1's expected value of choosing a is:

$$E\left[a\right] = 1 * p + 1 * (1-p) = 1$$

Player 1's expected value of choosing b is:

$$E\left[b\right] = 3 * p + 0 * (1-p) = 3p$$

As long as $E\left[a\right] \geq E\left[b\right]$ this will be a Bayes-Nash equilibrium. If

$$\begin{aligned} 1 &\geq 3p \\ \frac{1}{3} &\geq p \end{aligned}$$

then there is a pure strategy Bayes-Nash equilibrium. If $p \in \left[0, \frac{1}{3}\right]$ then Player 1 choose a, Player 2 (H type) choose x, and Player 2 (L type) choose y is a pure strategy Bayes-Nash equilibrium.

If Player 1 chooses b, Player 2's best response is y if H type and y if L type. Player 1's expected value of choosing b is:

$$E[b] = 2 * p + 0 * (1 - p) = 2p$$

Player 1's expected value of choosing a is:

$$E[a] = 1 * p + 1 * (1 - p) = 1$$

If $E[b] \geq E[a]$ this will be a Bayes-Nash equilibrium. If

$$\begin{aligned} 2p &\geq 1 \\ p &\geq \frac{1}{2} \end{aligned}$$

then there is a pure strategy Bayes-Nash equilibrium. To sum up:

If $p \in \left[\frac{1}{2}, 1\right]$ then Player 1 choose b, Player 2 (H type) choose y, and Player 2 (L type) choose y is a pure strategy Bayes-Nash equilibrium. Note that this equilibrium is the one in part **a** when $p = 0.75$.

If $p \in \left(\frac{1}{3}, \frac{1}{2}\right)$ then there is no pure strategy Bayes-Nash equilibrium.

If $p \in \left[0, \frac{1}{3}\right]$ then Player 1 chooes a, Player 2 (H type) choose x, and Player 2 (L type) choose y is a pure strategy Bayes-Nash equilibrium.