

SEQUENTIAL MEDIATION IN PARTNERSHIPS

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ABSTRACT. This paper studies the joint design of game forms and contracts in team moral hazard. The framework encompasses both simultaneous and sequential move games and generalizes static mediation in partnerships. Agents choose from finite action sets, and their action profile determines the distribution of publicly verifiable signals. Rather than restricting attention to simultaneous move games, the designer can freely choose and randomize over sequential move games where each agent chooses her own action once. We provide a necessary and sufficient condition on the signal structure that characterizes the set of all implementable action profiles under budget balance. In the class of binary-action coordination problems, we identify the least-cost bonus schemes and associated game forms to achieve coordination as either an equilibrium outcome or the unique equilibrium outcome. Most notably, the uniqueness requirement entails no additional cost under sequential mediation.

1. INTRODUCTION

In many economic environments, cooperation and joint production depend on how information, timing, and monetary transfers are structured within an organization. Start-ups, research collaborations, and project-based partnerships often rely on decentralized arrangements where agents must exert effort without direct supervision. Moral hazard is usually one of the most significant issues in such environments, and it is rarely only transfers that sustain cooperation; rather, the

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timing of moves and others' observability play equally important roles in shaping incentives. For example, in software development, large projects are divided into interdependent modules—front-end design, database integration, testing, debugging, etc. Each module may depend on others being partially completed, and the timing of development can itself be part of the design of the team production process: engineers decide when to start or deliver their part based on progress they observe elsewhere. Similar endogenous dynamics arise in a wide range of economic and organizational settings.

Despite their practical prevalence, such interactions within organizations are typically taken as exogenously given in models of team production. While it is a well-established theoretical insight that introducing uncertainty through a mediator (e.g. [Rahman \(2012\)](#), [Rahman and Obara \(2010\)](#)) can relax the incentive constraints for agents to work in the static team framework of [Holmstrom \(1982\)](#), much less is known about how such mechanisms operate in sequential production processes.

This paper addresses this gap by introducing sequentially mediated contracts in Section 2, a framework in which a disinterested mediator not only randomizes over transfers, as in [Rahman and Obara \(2010\)](#), but also designs the sequence of moves and the information revealed at each stage. The mediator can, for instance, privately select which agent acts first, or randomize over move orders. That is, timing itself is an instrument of incentive design under sequential mediation.

We are interested in the joint design of game forms and transfers under team moral hazard. To this end, we consider two concrete questions. The first one, studied in Section 4, concerns the design of budget-balanced sequentially mediated contracts in a general partnership problem, where the transfers can depend on a publicly verifiable signal whose distribution is determined by the action profile, and on the sequential-move game. The main result (Theorem 4.1) is a necessary

and sufficient condition on the signal structure, referred to as the indicating punishment property (IPP), under which a given action profile can be implemented as an equilibrium outcome. In sequential-move games, identifying deviations is less of a problem, and the key challenge is to correctly punish the deviator when a deviation occurs. IPP requires that, when an agent deviates, subsequent agents can credibly convey this fact through their corresponding actions to the contract, which then punishes the deviator based on the realized public signal.

The second question, examined in Section 5, focuses on binary-action coordination problems among multiple agents. In this setting, each agent can either work or shirk and is protected by limited liability, and the contractible outcome is either success or failure. Following the literature, the probability of success is an increasing and convex function of the number of agents who work. The principal acts as the mediator, whose problem is to choose the least-cost success-contingent bonus scheme and the associated sequential-move game under which all agents working is either an or the unique equilibrium outcome. Theorem 5.1 establishes that obfuscation is necessary by constructing an optimal contract under which agents are uncertain about their move orders, improving on the contract under the fully transparent sequential move game. Moving to unique implementation, Theorem 5.6 constructs an optimal sequentially mediated contract that again features opacity, although it randomizes differently over sequential moves and bonuses to ensure that working is the unique outcome surviving iterated elimination of strictly dominated strategies. Perhaps surprisingly, the uniqueness requirement does not entail any additional cost: the total cost of this contract is the same as that of the optimal partial implementation contract.

The main economic insight emerging from the analysis is the structure of peer monitoring in mediated sequential production. Peer monitoring typically takes the form of endogenous punishment: a worker who observes shirking upstream optimally reduces her own effort downstream, thereby lowering joint output and imposing a cost on the deviator. This pattern is prevalent in many settings (e.g.,

von Krogh et al. (2003), Bandiera et al. (2005), Mas and Moretti (2009)). In academic research teams, for instance, coauthors adjust continuation effort in response to perceived partner contributions (Azoulay et al. (2019)). Similarly, in open-source software projects, downstream contributors frequently respond to low-quality or delayed upstream inputs by withholding effort or exiting the project (Lerner and Tirole (2002)). Importantly, such environments rarely feature full transparency; instead, information is revealed selectively following problems, closely mirroring the mechanism uncovered in this paper. While transparency strengthens incentives by allowing agents to condition actions on observed effort, it also raises the cost of providing incentives, especially for late movers who can perfectly infer earlier behavior (Winter (2006)). Thus, the effectiveness of peer monitoring depends critically on how information and timing are structured through mediation, rather than on transparency per se, as the main findings illustrate.

1.1. Related Literature. The classic contracting literature following Holmström (1982) analyzes team moral hazard in static or simultaneous-move environments, where all agents choose effort simultaneously and incentives are provided through transfers (e.g., Legros and Matsushima (1991), Legros and Matthews (1993)). One central result is that budget imbalance is necessary to alleviate the free-riding problem in a symmetric partnership. Within the static framework, Rahman and Obara (2010), Rahman (2012), and Strausz (2012) further establish that mediation improves contractual ability, and budget balance can be restored by occasionally and privately recommending some agents to deviate from their target actions. While a smaller literature studies sequential partnerships (e.g. Strausz (1999) on budget balance under non-stochastic signal structures), the contracts are unmediated. As summarized in Table 1, this paper studies budget balanced contracts under sequential mediation and characterizes general public signal structures for an action profile to be implementable.

TABLE 1. Budget Balance (BB) in Team Moral Hazard

	Unmediated	Mediated
Static	Holmstrom (1982) : budget imbalance	Rahman and Obara (2010) : signals satisfy IOA for virtual implementation
Sequential	Strausz (1999) : non-stochastic signals	This paper : signals satisfy IPP

Starting from [Segal \(2003\)](#) and [Winter \(2004\)](#), the recent literature on binary-action team coordination problems has also considered both static and dynamic production processes. The focus here is on the principal’s least-cost bonus schemes. For the problem of (partially) implementing the outcome that all agents work as an equilibrium, the optimal unmediated static contract is quite straightforward, as summarized in the comprehensive survey by [Halac \(2025\)](#). For static mediation, one can apply the result in [Rahman \(2012\)](#) to characterize the virtually least cost contract. [Winter \(2006\)](#) considers unmediated contracts under sequential production and allows the principal to control what agents can observe about others’ effort choices. The main finding in [Winter \(2006\)](#) is that under supermodular signal structures, transparency benefits the principal as it grants peer monitoring. In contrast, this paper finds that the optimal contract with sequential mediation is opaque as it can save the cost of motivating late movers to work by introducing uncertainty in their move orders. This discussion is collected in Table 2.

For the more challenging problem of implementing all agents working as the unique equilibrium outcome, [Winter \(2004\)](#) considers the simultaneous-move game and public (unmediated) contracts. He finds that the optimal contract is discriminatory as all agents working is achieved by the process of iterated elimination of strictly dominated strategies (IESDS) under the contract. [Halac et al. \(2021\)](#) also

TABLE 2. Partial Implementation of Coordination

	Unmediated	Mediated
Static	Halac (2025)	Rahman (2012) : virtual implementation
Sequential	Winter (2006) : full transparency	This paper : obfuscation

study the simultaneous-move game but allow for private (mediated) contracts. They show that, by introducing rank uncertainty that generates mutual assurance in the iterated elimination process, the optimal private contract, improves on the public counterpart and is non-discriminatory. With sequential production processes and public (unmediated) contracts, the result in [Winter \(2010\)](#) implies that the optimal contract coincides with the one under partial implementation: a fully transparent sequential-move game and discriminatory bonuses.¹ In comparison, this paper shows that with sequential mediation, although the uniqueness requirement is no more costly than partial implementation and the optimal contract has the same game form, both the randomization probabilities and bonuses differ to address uniqueness via IESDS. Moreover, the optimal contract is only superficially discriminatory: while it treats identical agents differently, their expected payoffs are the same. Table 3 summarizes the findings.

More broadly, this paper contributes to the recent literature on sequential information design (e.g. [Doval and Ely \(2020\)](#) and [Makris and Renou \(2023\)](#)). [Doval and Ely \(2020\)](#) examine incomplete-information games where both the extensive

¹Also see [Cheng et al. \(2025\)](#) for the study of AI in sequential production building on the framework of [Winter \(2010\)](#).

TABLE 3. Full Implementation of Coordination

	Unmediated (Public)	Mediated (Private)
Static	Winter (2004) : IESDS; discriminatory	Halac et al. (2021) : IESDS and rank uncertainty; non-discriminatory
Sequential	Winter (2010) : full transparency; discriminatory	This paper : uncertainty in the order of moves; non-discriminatory

form and information structure are flexible.² They characterize the more admissible equilibrium outcomes that can arise in this setting without monetary transfers. By studying the joint design of information and contracts in team moral hazard problems without adverse selection, we obtain new insights on organizational structures and their economic implications.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 provides illustrative examples in two-agent partnerships. Section 4 characterizes the set of implementable action profiles under budget-balanced sequential mediation. Section 5 examines the principal's least-cost design problems under both partial and full implementation. Section 6 concludes.

2. MODEL

There is a finite set $\mathcal{I} = \{1, 2, \dots, N\}$ of agents and each agent i can take an action a_i in a finite set A_i . Let $A = \prod_{i \in \mathcal{I}} A_i$ be the set of all action profiles. Let Y be a finite set of publicly verifiable signals. Denote by $\mu : A \rightarrow \Delta(Y)$ the conditional distribution of public signals. That is, for each action profile $a = (a_i)_{i \in \mathcal{I}} \in A$, the probability distribution of the public signal is $\mu(\cdot | a) \in \Delta(Y)$. For example, Y can

²See also [Nishihara \(1997\)](#) and [Salcedo \(2017\)](#) for earlier studies of complete-information games.

be the set of possible outputs generated by the agents. Agents have quasi-linear utility functions. For each $i \in \mathcal{I}$, if the action profile a is chosen and agent i pays a monetary transfer $t_i \in \mathbb{R}$, then agent i 's payoff is $v_i(a) - t_i$, where $v_i : A \rightarrow \mathbb{R}$ is agent i 's payoff function from the action profiles. A *transfer schedule* is a mapping $t : \mathcal{I} \times Y \rightarrow \mathbb{R}$, where $t_i(y)$ is the transfer agent $i \in \mathcal{I}$ receives when the realized outcome is $y \in Y$. Let T denote the set of *budget-balanced* transfer schedules, that is, $t \in T$ if $\sum_{i \in \mathcal{I}} t_i(y) = 0$ for all $y \in Y$.

Instead of assuming that agents play a simultaneous game, we consider all possible extensive form games in which each agent chooses her action once. In particular, it is possible that one agent observes some other agents' actions before choosing her own in some extensive form games. Let $\Gamma(A)$ denote the set of all possible extensive form games in which agents choose their actions sequentially and each agent chooses her action once. We assume that there is a hypothetical and disinterested mediator who chooses both the extensive form game and the corresponding compensation schedule. In particular, the compensation schedule can depend on the chosen extensive form game. Importantly, by selecting both the extensive form game and the transfer schedule randomly and combining different nodes of the same agent into an information set, it is possible that an agent is uncertain about the realized extensive form game, namely, whether some other agents move before or after her, and the realized transfer schedule, although she will make inference using Bayes rule given the mediator's selection rule.

Formally, a *sequentially mediated contract* (π, H) consists of a probability distribution π over the product set of extensive form games and transfer schedules $\Gamma(A) \times T$, and a collection of partitions of agents' nodes, $H = (H_i)_{i \in \mathcal{I}}$, where for each agent i the corresponding partition H_i consists of her information sets in the mediated extensive-form game. Note that $\pi \in \Delta(\Gamma(A) \times T)$ and H induce an extensive-form game with perfect recall. For each $i \in \mathcal{I}$, a behavioral strategy for agent i is a map $\sigma_i : H_i \rightarrow \Delta(A_i)$, where $\sigma_i(a_i|h_i)$ is the probability that agent i chooses $a_i \in A_i$ at her information set $h_i \in H_i$. A strategy profile $\sigma^* = (\sigma_i^*)_{i \in \mathcal{I}}$ is

a subgame perfect equilibrium if its continuation play at any subgame is a Nash equilibrium. Every subgame perfect equilibrium induces a distribution over action profiles.

3. EXAMPLES

Consider the following two-agent partnership with binary actions and symmetric public signals. Each agent $i = 1, 2$ can either exert effort (i.e., work) $a_i = 1$ or shirk $a_i = 0$. Let $c > 0$ be each agent's effort cost. The outcome can be either success, denoted by g , or failure, denoted by b . The probability of g is $P(a_1 + a_2)$, where $P(\cdot)$ is a function of the sum of both agents' efforts. Agent i 's utility function is $t_i - ca_i$, where $t_i \in \mathbb{R}$ is the transfer to agent i .

In this example, the set of extensive form games $\Gamma(A)$ consists of three elements: a simultaneous move game, a sequential move game in which agent 1 moves first, and a sequential move game in which agent 2 moves first.

3.1. Budget-balanced Partnership. First consider the partnership moral hazard problem without a principal. We will characterize the public signal structure under which the action profile $a = (a_1, a_2) = (1, 1)$ can be implemented by sequentially mediated contracts that satisfy budget balance. The set of all budget-balanced transfer schedules is

$$T = \{(t_1, t_2) : t_i : \{g, b\} \rightarrow \mathbb{R}, \forall i = 1, 2, \text{ and } t_1(g) + t_2(g) = t_1(b) + t_2(b) = 0\}$$

A sequentially mediated and budget-balanced contract is a probability distribution $\pi \in \Delta(\Gamma(A) \times T)$ together with the agents' information sets.

Assume that $P(1) \neq P(0)$. That is, the probabilities of a good outcome differ when either only one or no agent exerts effort. Consider the sequential contract in Figure 1, where $\pi = 1/2$, $t_1^l(g) = t_2^l(g) = t_1^r(g) = t_2^r(g) = 0$, and $t_1^l(b) = -t_2^l(b) = t_2^r(b) = -t_1^r(b) = -2c/(P(1) - P(0))$. Note that in principle the contract can involve

random transfer schedules conditional on the extensive form game; however, because the agents are risk neutral, it is without loss to consider a single transfer schedule for each game.

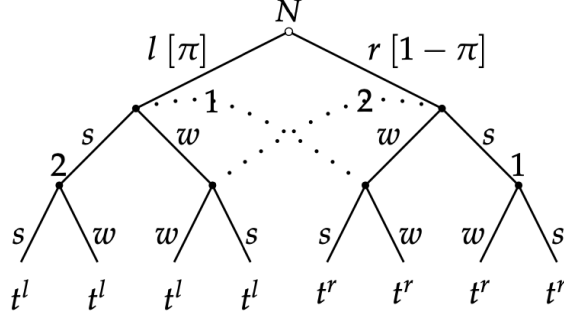


FIGURE 1. Sequential Mediation

Under this sequentially mediated contract, there is a subgame perfect equilibrium with outcome $a^* = (a_1^*, a_2^*) = (1, 1)$ with probability one, that is, both agents exert effort. In addition, if the agent who moves first does not exert effort, then the other agent will also shirk. To see this, consider the incentive constraints in the game l chosen by nature (the game r is symmetric). For agent 2, if she observes that agent 1 has chosen s , then she learns that the game is l and thus we have

$$\begin{aligned} P(0)t_2^l(g) + (1 - P(0))t_2^l(b) &= (1 - P(0)) \cdot \frac{2c}{P(1) - P(0)} \\ &\geq (1 - P(1)) \cdot \frac{2c}{P(1) - P(0)} - c = P(1)t_2^l(g) + (1 - P(1))t_2^l(b) - c. \end{aligned}$$

For agent 1, given that agent 2 chooses w in the game r under the specified strategy profile, she believes that she is in game l with probability one half and thus we have

$$\begin{aligned} &\frac{1}{2} [P(2)t_1^l(g) + (1 - P(2))t_1^l(b)] + \frac{1}{2} [P(2)t_1^r(g) + (1 - P(2))t_1^r(b)] \\ &= -\frac{1 - P(2)}{2} \cdot \frac{2c}{P(1) - P(0)} + \frac{1 - P(2)}{2} \frac{2c}{P(1) - P(0)} \\ &\geq -\frac{1 - P(0)}{2} \cdot \frac{2c}{P(1) - P(0)} + \frac{1 - P(1)}{2} \frac{2c}{P(1) - P(0)} \\ &= \frac{1}{2} [P(0)t_1^l(g) + (1 - P(0))t_1^l(b)] + \frac{1}{2} [P(1)t_1^r(g) + (1 - P(1))t_1^r(b)]. \end{aligned}$$

Therefore, we have verified that this strategy profile, in which the on-path outcome is $a_1 = a_2 = 1$, is a subgame perfect equilibrium.

It is instructive to compare the above *sequentially* mediated contract with the *statically* mediated contract in [Rahman and Obara \(2010\)](#), which can be viewed as another extensive-form game with nature's moves and the corresponding transfer schedules in Figure 2.

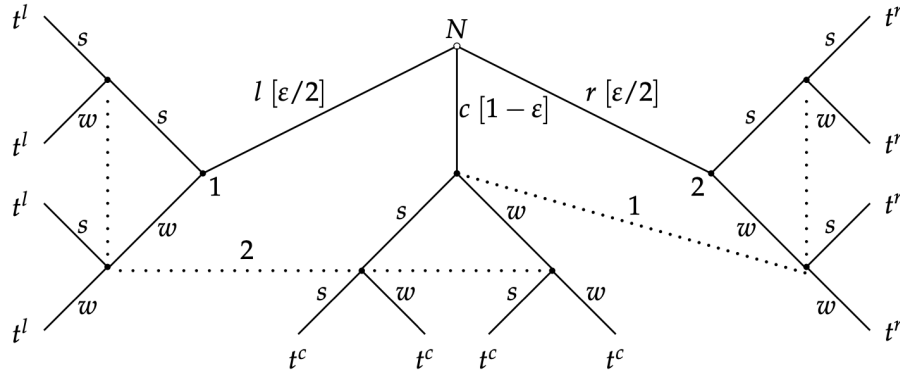


FIGURE 2. Static Mediation.

[Rahman and Obara \(2010\)](#) show that the action profile $a^* = (1, 1)$ can be virtually implemented in the above extensive-form game when $P(2) > P(1) > P(0)$: for any $\varepsilon > 0$, under the following transfers (t^l, t^c, t^r) ,

$$t_1^l(g) = -t_2^l(g) = -\frac{(2 - \varepsilon)c}{\varepsilon(P(1) - P(0))}, \quad t_1^l(b) = t_2^l(b) = 0,$$

$$t_2^r(g) = -t_1^r(g) = -\frac{(2 - \varepsilon)c}{\varepsilon(P(1) - P(0))}, \quad t_1^r(b) = t_2^r(b) = 0,$$

and

$$t_1^c = t_2^c \equiv 0.$$

$a^* = (1, 1)$ is played with probability $1 - \varepsilon$ in equilibrium. In particular, (i) both agents work when nature chooses c , (ii) agent 1 shirks and agent 2 works when nature chooses l , (iii) agent 2 shirks and agent 1 works when nature chooses r .

This comparison highlights the differences between sequential and static mediations. First, in the static contract, on-path punishment is necessary to provide incentives and balance the budget. In contrast, the punishment for shirking occurs only off-path in the sequential contract, which leads to full implementation of the action profile a^* . Second, the punishment transfers t^l and t^r in the static contract need to grow unbounded as ε decreases to zero, whereas the transfers are bounded under the sequential contract. Finally, the underlying mechanisms of the two contracts are different. The static contract is based on identifying and rewarding the non-deviating agent: whenever nature chooses r , because agent 1 shirks, she will be *rewarded* if the bad outcome happens.³ The sequential contract relies on peer monitoring and punishing the deviator: if an agent shirks, then with probability one half she will be caught by the other agent, in which case the other agent also shirks to punish the first agent.

Finally, we show that when $P(1) = P(0)$, $a^* = (1, 1)$ cannot be implemented, even virtually; furthermore, in this case the only implementable action profile is $(0, 0)$.

Suppose by contradiction that there is a sequentially mediated contract under which $a^* = (1, 1)$ is an equilibrium outcome. On the equilibrium path, let q denote the probability that agent 1 moves before agent 2 and $1 - q$ the probability that agent 1 moves after agent 2. Let $\bar{t} = (\bar{t}_1, \bar{t}_2)$ denote the agents' *expected* transfers when agent 1 moves before agent 2 and $\hat{t} = (\hat{t}_1, \hat{t}_2)$ the agents' *expected* transfers when agent 1 moves after agent 2.

On the equilibrium path, the incentive compatibility constraint for agent 1 is

$$\begin{aligned} & q[P(2)\bar{t}_1(g) + (1 - P(2))\bar{t}_1(b)] + (1 - q)[P(2)\hat{t}_1(g) + (1 - P(2))\hat{t}_1(b)] - c \\ & \geq q[P(0)\bar{t}_1(g) + (1 - P(0))\bar{t}_1(b)] + (1 - q)[P(1)\hat{t}_1(g) + (1 - P(1))\hat{t}_1(b)], \end{aligned}$$

³This is at the cost of agent 2; but in the case with more than two agents, at most one player shirks in the static contract of [Rahman and Obara \(2010\)](#), and the bad outcome indicates that she does not deviate so that the contract will reward her at the expense of all others.

where we have used the assumption that $P(1) = P(0)$ for the payoff on the right hand side. This inequality can be simplified as

$$q[P(2) - P(0)][\bar{t}_1(g) - \bar{t}_1(b)] + (1 - q)[P(2) - P(0)][\hat{t}_1(g) - \hat{t}_1(b)] \geq c.$$

Similarly, agent 2's incentive compatibility constraint on the equilibrium path is

$$q[P(2) - P(0)][\bar{t}_2(g) - \bar{t}_2(b)] + (1 - q)[P(2) - P(0)][\hat{t}_2(g) - \hat{t}_2(b)] \geq c.$$

Since budget balance implies that

$$\bar{t}_1(\cdot) + \bar{t}_2(\cdot) = \hat{t}_1(\cdot) + \hat{t}_2(\cdot) = 0,$$

summing up the two incentive compatibility constraints, we obtain $0 \geq 2c$, a contradiction.

3.2. Least-Cost Coordination. Now we drop budget balance and consider the problem with a principal who optimizes over the set of all sequentially mediated contracts subject to limited liability to implement $a = (1, 1)$. Thus, the principal chooses the least-cost success-contingent bonus schemes and the corresponding extensive-form games jointly. Here we further assume for simplicity that $P(0) = \alpha^2$, $P(1) = \alpha$, and $P(2) = 1$, where $\alpha \in (0, 1)$, as in [Winter \(2006\)](#) and [Halac \(2025\)](#).

First consider the criterion of (partially) implementing $a = (1, 1)$ as a subgame perfect equilibrium outcome. The mediated bonus contract in [Figure 3](#), where $\pi = 1/2$, $b_1^l = b_2^r = 2c/(1 - \alpha^2)$, and $b_2^l = b_1^r = 0$, is optimal. Under this contract, both agents exert effort in the information set after Nature's move and shirk at the other singleton information set, since the following incentive constraints are satisfied:

$$\frac{b_1^l}{2} + \frac{b_1^r}{2} - c \geq \frac{\alpha^2 b_1^l}{2} + \frac{\alpha b_1^r}{2}, \quad \frac{b_2^l}{2} + \frac{b_2^r}{2} - c \geq \frac{\alpha b_2^l}{2} + \frac{\alpha^2 b_2^r}{2},$$

and

$$\alpha^2 b_2^l \geq \alpha b_2^l - c, \quad \alpha^2 b_1^r \geq \alpha b_1^r - c.$$

Moreover, the expected cost of this contract to the principal is $2c/(1 - \alpha^2)$. To see that this is optimal, note that $1 - \alpha^2$ is the maximal change in the success probability and thus $c/(1 - \alpha^2)$ is the least expected bonus amount to motivate an agent to work.

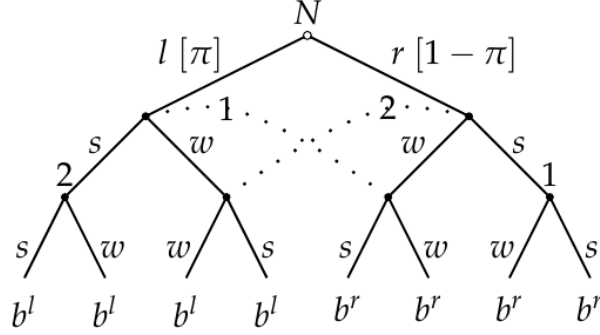


FIGURE 3. Optimal Least-cost Coordination.

The fact that the above least-cost mediated contract features obfuscation does not contradict the main finding in Winter (2010), which establishes that transparency among agents benefits the principal in the presence of effort complementarity. The difference is that Winter (2010) fixes the sequential move game and examines the corresponding optimal unmediated contract, whereas here we jointly design the game and contract. Intuitively, when agents move sequentially and the game is fully transparent, the principal needs to incentivize the second mover, who perfectly observes the first mover's effort choice, with a larger bonus. By introducing uncertainty to the order of moves, the optimal mediated contract reduces this cost.

Next consider the criterion of (fully) implementing $a = (1, 1)$ as the unique subgame perfect equilibrium outcome. For the mediated contract in Figure 3, it is straightforward to verify that there is another subgame perfect equilibrium in which agents always shirk at all their information sets.

One way to eliminate such a coordination failure is to consider fully transparent sequential-move games, as in Figure 4, where $b_1 = c/(1 - \alpha^2)$ and $b_2 = c/(1 - \alpha)$. Indeed, a key insight from the seminal work of Winter (2004) is that, when an agent is uncertain about the other agent's action, a larger bonus is needed to make working a dominant action for him. Full transparency gets rid of this uncertainty and even outperforms the optimal private contract with simultaneous-move games in Halac et al. (2021) (which are equivalent to contracts under static mediation in our language), which costs the principal $4c/(1 - \alpha^2) (> c/(1 - \alpha^2) + c/(1 - \alpha))$.

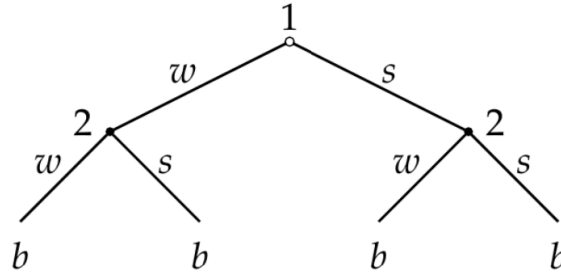


FIGURE 4. Fully Transparent Coordination.

Nonetheless, the fully transparent sequential-move game is suboptimal. In fact, the optimal contract still features obfuscation as the one in Figure 3, with different randomization probability and bonuses:

$$\pi = \frac{\alpha}{1 + \alpha}, \quad b_1^l = \frac{c}{\alpha(1 - \alpha)} + \varepsilon, \quad b_2^r = \frac{c}{(1 - \pi)(1 - \alpha^2)} + \varepsilon, \quad b_2^l = b_1^r = 0,$$

for some arbitrarily small $\varepsilon > 0$.

To see that both agents working is a unique subgame perfect equilibrium outcome, first note that after the first mover shirks, the other agent is informed and infers that his bonus is zero, so it is strictly optimal for him to shirk too. Given this,

the normal form of the reduced extensive-form game becomes

	w	s
w	$\frac{\alpha^2 c}{1-\alpha^2} + \pi\varepsilon, \frac{\alpha^2 c}{1-\alpha^2} + (1-\pi)\varepsilon$	$\frac{\alpha^2 c}{1-\alpha^2} + \alpha\pi\varepsilon, \frac{\alpha^2 c}{1-\alpha^2} + \alpha^2(1-\pi)\varepsilon$
s	$\frac{\alpha^2 c}{1-\alpha^2} + \alpha^2\pi\varepsilon, \frac{(2\alpha-1)c}{1-\alpha^2} + \alpha(1-\pi)\varepsilon$	$\frac{\alpha^2 c}{1-\alpha^2} + \alpha^2\pi\varepsilon, \frac{\alpha^2 c}{1-\alpha^2} + \alpha^2(1-\pi)\varepsilon$

where working is a strictly dominant strategy for agent 1 (not for agent 2 though), and given agent 1 works, it is strictly optimal for agent 2 to work too. Thus, both agents working is the unique outcome that survives iterated elimination of strictly dominated strategies in the normal form.

It is worth emphasizing that the expected cost of the above contract to the principal is $\pi b_1^l + (1-\pi)b_2^r = 2c/(1-\alpha^2)$, which is the same as the minimal cost under partial implementation. That is, the uniqueness requirement does not impose any additional cost on the principal under sequential mediation. Furthermore, despite that this contract appears discriminatory—identical agents are treated differently in both the game form and the ex post bonuses, the expected bonuses of both agents are the same. These results are in sharp contrast to the findings in [Halac et al. \(2021\)](#) under static mediation.

4. ANALYSIS: BUDGET BALANCE

This section characterizes the public signal structure $\mu : A \rightarrow \Delta(Y)$ under which an action profile $a = (a_i)_{i=1}^N \in A$ is implementable by budget-balanced sequentially mediated contracts. For results here, we will focus on the following subclass of contracts, which keep agents minimally informed when choosing their actions on path and maximally informed after some deviation occurs.

Formally, a *canonical sequentially mediated contract* consists of a probability distribution π over N sequential move games, $\gamma^i \in \Gamma(A)$ in which agent i chooses her action first for all i , and N transfer schedules $t^i = (t_n^i)_{n=1}^N$ for all $i \in \mathcal{N}$. Furthermore, in the game γ^i , if agent i plays any action $a'_i \neq a_i$, her choice is revealed to all other agents publicly; if agent i plays a_i , then others are uninformed. That is, from any

agent i 's perspective, there is an information set of her that contains the following her decision nodes: the initial node of γ^i , the node that follows a_j in γ^j for all $j \neq i$; moreover, in all other information sets of agent i , she observes deviations from some agent and this fact is common among all agents. The mediated contract in Figure 1 of Section 3 is an example.

Intuitively, an agent, together with a transfer schedule, is chosen randomly by nature to be monitored by others in a sequentially mediated contract. Furthermore, each agent is unsure whether she is being monitored in her information set immediately following nature's choice. When an agent i is monitored, if she chooses a_i , then other agents do not observe this choice; however, if she chooses any $a'_i \neq a_i$, others are informed and can coordinate their actions depending on a'_i .

An action profile $a \in A$ is implementable by a budget-balanced canonical sequentially mediated contract if there exist (i) $\pi = (\pi_i)_{i \in \mathcal{N}}$ where $\pi_i \geq 0$ and $\sum_{i \in \mathcal{N}} \pi_i = 1$, (ii) $t = (t^1, \dots, t^N)$ where $t^i = (t_n^i)_{n \in \mathcal{N}}$ for each i and $t_n^i : Y \rightarrow \mathbb{R}$ for each n , and (iii) $\beta = (\beta_{-i})_{i \in \mathcal{N}}$ where $\beta_{-i} : A_i \rightarrow A_{-i}$, such that for each $i \in \mathcal{N}$ and $a'_i \in A_i$,

$$\begin{aligned}
 (1) \quad & \sum_{n \neq i} \pi_n \sum_{y \in Y} [v_i(a) - t_n^i(y)] \mu(y|a) + \pi_i \sum_{y \in Y} [v_i(a) - t_i^i(y)] \mu(y|a) \\
 & \geq \sum_{n \neq i} \pi_n \sum_{y \in Y} [v_i(a'_i, a_{-i}) - t_n^i(y)] \mu(y|a'_i, a_{-i}) \\
 & \quad + \pi_i \sum_{y \in Y} [v_i(a'_i, \beta_{-i}(a'_i)) - t_i^i(y)] \mu(y|a'_i, \beta_{-i}(a'_i))
 \end{aligned}$$

and for each $i \in \mathcal{N}$ and $y \in Y$,

$$(2) \quad \sum_{n=1}^N t_n^i(y) = 0.$$

Note that the probability distribution π in the sequentially mediated contract is in fact redundant as we can redefine t_n^i to be $\pi_n t_n^i$, thus the incentive compatibility

constraints (1) can be rewritten as:

$$(3) \quad \sum_{n \neq i} \sum_{y \in Y} [\mu(y|a'_i, a_{-i}) - \mu(y|a)] t_i^n(y) + \sum_{y \in Y} [\mu(y|a'_i, \beta_{-i}(a'_i)) - \mu(y|a)] t_i^i(y) \\ \geq (N-1)[v_i(a'_i, a_{-i}) - v_i(a)] + [v_i(a'_i, \beta_{-i}(a'_i)) - v_i(a)].$$

The right-hand side of (3) is the expected payoff gain for agent i from playing a'_i instead of a_i , depending on whether nature chooses the extensive game to monitor agent i . The left-hand side of (3) is the expected increment in agent i 's transfer from playing a'_i instead of a_i , again depending on nature's choice.

A behavioral strategy profile $(\alpha_1, \dots, \alpha_N)$ in the canonical sequential mediated contract is said to be *disobedient* if there exists $n \in \mathcal{I}$ such that $\alpha_n(a_n) < 1$ at agent n 's information set immediately after nature's move.

We say that the public signal structure $\mu : A \rightarrow \Delta(Y)$ satisfies the *indicating punishment property (IPP)* at $a \in A$ if there exists $\beta = (\beta_{-i})_{i \in \mathcal{N}}$ with $\beta_{-i}(a_i) = a_{-i}$ for all i such that for any disobedient strategy profile $(\alpha_1, \dots, \alpha_N)$, there exists $i, j \neq i$, and y such that

$$(4) \quad \sum_{a'_i} \alpha_i(a'_i) \mu(y|a'_i, \beta_{-i}(a'_i)) \neq \sum_{a'_j} \alpha_j(a'_j) \mu(y|a'_j, a_{-j}).$$

Intuitively, IPP at $a = (a_i)_{i \in \mathcal{I}}$ means that when an agent i deviates from a_i , other agents can "indicate" this deviation to the contract via the strategy profile β , so that agent i can be punished accordingly. Moreover, such an indication must be "credible," in the sense that it cannot itself be interpreted as a unilateral deviation by some other agent $j \neq i$. For the example in Section 3.1, IPP at (w, w) holds if and only if $P(1) \neq P(0)$. To see this, let $\beta_{-i}(s) = s$ and $\beta_{-i}(w) = w$ for $i = 1, 2$. Without loss of generality, suppose agent 1 is disobedient, i.e., shirking at her first information set. If agent 2 is obedient, then (4) is equivalent to the condition that $P(0) \neq P(2)$ or $P(2) \neq P(1)$, which is equivalent to $P(1) \neq P(0)$; if agent 2 is disobedient, then (4) is equivalent to $P(0) \neq P(1)$.

Theorem 4.1 below establishes that IPP characterizes implementability by canonical budget-balanced sequentially mediated contracts.

Theorem 4.1. *A strategy profile $a \in A$ is implementable by a budget-balanced canonical sequentially mediated contract for all payoff functions $v = (v_i)$ if and only if the public signal structure satisfies the indicating punishment property at $a \in A$.*

Proof. For sufficiency, we build on the proof idea in Rahman and Obara (2010) and prove the contrapositive statement as follows. First fix any β . We can rewrite the incentive compatibility constraints (3) as follows: for each i and $a'_i \in A_i$,

$$(5) \quad \sum_{n \neq i} \sum_{y \in Y} [\mu(y|a'_i, a_{-i}) - \mu(y|a)] t_i^n(y) + [\mu(y|a'_i, \beta_{-i}(a'_i)) - \mu(y|a)] t_i^i(y) \geq \Delta^i(a'_i),$$

where $\Delta^i(a'_i) = 0$ if $a'_i = a_i$, and $\Delta^i(a'_i) = 1$ if $a'_i \neq a_i$.

Consider the problem of choosing t to minimize 0 subject to (2) and (5). Let $\eta^i(y)$ be the Lagrangian multiplier associated with (2) and $\lambda^i(a'_i)$ the Lagrangian multiplier associated with (5). Then the dual problem is to choose $\eta = (\eta^i(y))_{i \in \mathcal{N}, y \in Y}$ and $\lambda = (\lambda^i(a'_i))_{i \in \mathcal{N}, a'_i \in A_i}$, where $\lambda^i(a'_i) \geq 0$ for all a'_i and i , to maximize

$$\sum_{i \in \mathcal{N}} \sum_{a'_i \in A_i} \lambda^i(a'_i) \Delta^i(a'_i)$$

subject to

$$(6) \quad \sum_{a'_n \in A_n} \lambda^n(a'_n) [\mu(y|a'_n, a_{-n}) - \mu(y|a)] = \eta^i(y), \quad \text{for all } n \neq i \text{ and } y,$$

and

$$(7) \quad \sum_{a'_i \in A_i} \lambda^i(a'_i) [\mu(y|a'_i, \beta_{-i}(a'_i)) - \mu(y|a)] = \eta^i(y), \quad \text{for all } y,$$

for all $i \in \mathcal{N}$.

By strong duality, there does not exist t satisfying (2) and (5) if and only if there is a feasible solution of the dual problem such that $\lambda^i(a'_i) > 0$ for some $i \in \mathcal{N}$ and

$a'_i \neq a_i$. Let $\Lambda \equiv \max_j \sum_{a'_j \in A_j} \lambda^j(a'_j) > 0$ and for all $j \in \mathcal{N}$, define

$$\alpha_j(a'_j) \equiv \begin{cases} \lambda^j(a'_j) / \Lambda, & \text{if } a'_j \neq a_j \\ 1 - \sum_{a'_j \neq a_j} \lambda^j(a'_j) / \Lambda, & \text{if } a'_j = a_j. \end{cases}$$

Then $(\alpha_1, \dots, \alpha_N)$ is disobedient. However, for all $i, j \in \mathcal{N}$ with $i \neq j$ and all $y \in Y$, we have

$$\sum_{a'_i} \alpha_i(a'_i) \mu(y|a'_i, \beta_{-i}(a'_i)) = \sum_{a'_j} \alpha_j(a'_j) \mu(y|a'_j, a_{-j}).$$

That is, IPP fails.

For necessity, suppose IPP fails. That is, for any $\beta = (\beta_{-i})_{i \in \mathcal{N}}$, there exists a disobedient strategy profile $(\alpha_1, \dots, \alpha_N)$ such that for some $i \in \mathcal{N}$ and $a'_i \neq a_i$, $\alpha_i(a'_i) > 0$, and for all $i, j \in \mathcal{N}$ with $i \neq j$ and $y \in Y$,

$$(8) \quad \sum_{a'_i} \alpha_i(a'_i) \mu(y|a'_i, \beta_{-i}(a'_i)) = \sum_{a'_j} \alpha_j(a'_j) \mu(y|a'_j, a_{-j}).$$

We will derive a contradiction if the strategy profile a is implementable by a sequentially mediated contract for all payoff functions. In particular, consider v given by

$$v_i(a) = 0, \quad v_i(a'_i, a_{-i}) = 1, \quad \text{for all } a'_i \neq a_i, \text{ and for all } i.$$

Suppose that a is implementable for v , that is, there exist β and t such that for each $i \in \mathcal{N}$ and $a'_i \in A_i$ with $a'_i \neq a_i$,

$$(9) \quad \sum_{n \neq i} \sum_{y \in Y} [\mu(y|a'_i, a_{-i}) - \mu(y|a)] t_i^n(y) + \sum_{y \in Y} [\mu(y|a'_i, \beta_{-i}(a'_i)) - \mu(y|a)] t_i^i(y) \geq 1.$$

Since IPP fails, there exists a disobedient α for which (8) holds. Then from inequality (9) we have that for all i ,

$$(10) \quad \begin{aligned} & \sum_{n \neq i} \sum_{y \in Y} \sum_{a'_i} \alpha_i(a'_i) [\mu(y|a'_i, a_{-i}) - \mu(y|a)] t_i^n(y) \\ & + \sum_{y \in Y} \sum_{a'_i} \alpha_i(a'_i) [\mu(y|a'_i, \beta_{-i}(a'_i)) - \mu(y|a)] t_i^i(y) \geq 1. \end{aligned}$$

However, (8) and (10) imply that

$$(11) \quad \sum_{n \neq i} \sum_{y \in Y} \sum_{a'_n} \alpha_n(a'_n) [\mu(y|a'_n, \beta_{-n}(a'_n)) - \mu(y|a)] t_i^n(y) \\ + \sum_{y \in Y} \sum_{a'_i} \alpha_i(a'_i) [\mu(y|a'_i, \beta_{-i}(a'_i)) - \mu(y|a)] t_i^i(y) \geq 1.$$

Summing up the inequalities (11) across all i yields

$$(12) \quad \sum_{i \in \mathcal{N}} \left[\sum_{y \in Y} \sum_{a'_i} \alpha_i(a'_i) [\mu(y|a'_i, \beta_{-i}(a'_i)) - \mu(y|a)] \sum_{n \in \mathcal{N}} t_n^i(y) \right] \geq 1,$$

which is impossible since $\sum_{n \in \mathcal{N}} t_n^i(y) = 0$ for all y and i . Therefore, we have reached a contradiction. $\square$

Remark 4.2. Because of the flexibility of choosing extensive-form games under sequential mediation, the main issue here is not whether deviations can be detected anymore. Instead, because under the contract agents lack additional protocols to verify deviations when they occur, they can only do so with their own effort choices. IPP essentially guarantees that agents can credibly verify who is the deviator through the public signals. An important requirement of IPP is that the strategy profile β applies uniformly to all disobedient strategy profiles, following the duality argument in the proof.

Remark 4.3. Compared to the virtual implementation under static mediation in [Rahman and Obara \(2010\)](#), an additional economic insight from sequential mediation is that punishment now only occurs off the equilibrium path by design, which avoids the issue in static mediation where the size of transfers grows to infinity as the probability that all agents work converges to one.

5. ANALYSIS: LEAST-COST COORDINATION

This section examines the problem of a principal who motivates the agents to exert effort to complete a project by designing a sequentially mediated contract, i.e., choosing both the sequential move game and the transfer schedule.

Assume that for each agent $i \in \mathcal{I}$, $A_i = \{0, 1\}$, where $a_i = 1$ denotes working and $a_i = 0$ denotes shirking. The cost of working for agent i is $c_i > 0$ and shirking incurs zero cost. Without loss of generality, we assume that $c_1 \geq c_2 \geq \dots \geq c_N$. The contractible signal space is $Y = \{g, b\}$, where g denotes success and b denotes failure. The probability of success depends on the number of agents who work, which is given by $P : \{0, 1, \dots, N\} \rightarrow [0, 1]$, a strictly increasing and strictly convex function.⁴ The workers have limited liability, and thus the principal's problem is to choose a probability distribution over (i) a bonus schedule $b = (b_i)_{i \in \mathcal{I}}$, where $b_i \geq 0$ is the payment to agent i when the realized signal is g , and (ii) a sequential-move game in $\Gamma(A)$ to induce all agents to work. Given an action profile $a = (a_j)_{j \in \mathcal{I}}$ and a bonus b_i , the payoff of agent i is $U_i(a, b_i) = P(|j \in \mathcal{I} : a_j = 1|)b_i - a_i c_i$.

5.1. Partial Implementation. First suppose that the principal can select her favorite equilibrium in the game induced by the sequentially mediated contract played by the agents. Formally, the principal's problem is

$$\min_{\pi \in \Delta(\Gamma(A) \times \mathbb{R}_+^N)} P(N) \sum_k \pi(b^k) \sum_{i \in \mathcal{I}} b_i^k$$

subject to the constraint that $a = (a_i)_{i \in \mathcal{I}} = (1, \dots, 1)$ is a subgame perfect equilibrium outcome of the game induced by π , where $\pi(b^k)$ is the marginal probability of the bonus schedule b^k .

The following result (Theorem 5.1) generalizes the first part of the example in Section 3.2 and shows that there is uncertainty in the order of moves and bonuses in the optimal contract; furthermore, in case of success only one agent gets the realized bonus.

Theorem 5.1. *There exists an optimal sequentially mediated contract, which randomizes over N pairs of sequential-move games and bonus schedules, $(\gamma^i, b^i)_{i \in \mathcal{I}}$,*

⁴The results in this section continue to hold when P is a more general function of the subsets of agents and P is strictly increasing and strictly supermodular.

with equal probability, where in each γ^i , agent i moves first and his action is revealed to all other agents only when he chooses $a_i = 0$, and $b^i = (b_j^i)_{j \in \mathcal{I}}$ is given by $b_i^i = Nc_i / (P(N) - P(0))$ and $b_j^i = 0$ for all $j \neq i$. Moreover, the expected cost to the principal in the optimal contract is $P(N) \sum_{i \in \mathcal{I}} c_i / (P(N) - P(0))$.

Proof. We begin by verifying that the strategy profile in which each agent i chooses $a_i = 1$ on the equilibrium path and $a_i = 0$ at all off-path information sets in a subgame perfect equilibrium in the game induced by the sequentially mediate contract. First, at any off-path information set, namely, after shirking by some predecessor, agent i is informed the corresponding sequential move game and bonus scheme in which his bonus is zero; thus, he will shirk as well. Next, at the on-path information set, agent i 's expected payoff from working is

$$\frac{P(N)}{N} b_i^i - c_i = \frac{P(0) c_i}{P(N) - P(0)},$$

which is weakly greater than his expected payoff from shirking, which is

$$\frac{P(0)}{N} b_i^i = \frac{P(0) c_i}{P(N) - P(0)}.$$

We then argue that this contract achieves the lowest cost for the principal among all feasible contracts. Since the payment scheme can only depend on success or failure, for different action profiles the maximal gap in the probability of success is $P(N) - P(0)$. Since agents are protected by limited liability, to incentivize agent i to work, the least success-contingent bonus that he needs to receive is $c_i / (P(N) - P(0))$. Therefore, the expected total bonus that the principal needs to pay is at least $P(N) \sum_{i \in \mathcal{I}} c_i / (P(N) - P(0))$, which is reached by the above sequentially mediated contract. $\square$

Remark 5.2. The randomization probabilities in the above optimal contract can in fact be arbitrary as long as they are all positive. If the sequential-move game γ^i is chosen with probability $\pi^i > 0$, then the corresponding positive bonus is given by $b_i^i = c_i / [\pi^i (P(N) - P(0))]$. The principal's total cost remains the same across all such contracts.

Remark 5.3. In the above mediated contract, it is important that the bonus scheme depends on the sequential-move game. In particular, only the agent being monitored by peers receives a positive bonus if the project succeeds. In addition, on the equilibrium path, agents are uncertain about the order of their moves and no information is revealed when no agent shirks, which dampens their incentive to shirk and thus reduces the bonuses relative to those in which the sequential-move game is fully transparent.

Remark 5.4. Similar to the observation in [Winter \(2010\)](#), here the requirement that all other agents observe any given agent's shirking in the optimal contract can be significantly weakened. If the first mover shirks, as long as each subsequent agent can see the effort choice of his immediate predecessor, the above mediate contract is still optimal, since one agent's shirking will cause a sequence of shirking by all subsequent agents under the bonus schedule.

The following result (Corollary 5.5) is an immediate implication of Theorem 5.1, which states that agents who face the same effort cost are treated symmetrically in expectation in the optimal mediated contract.

Corollary 5.5. *The optimal sequentially mediated contract in Theorem 5.1 is non-discriminatory: if $c_i = c_j = c$ for agents i and $j \neq i$, then their expected bonuses are the same, which equal $P(N)c / (P(N) - P(0))$.*

5.2. Full Implementation. Next consider the case in which the principal needs to guarantee all agents working as the unique equilibrium outcome. The principal's problem now is

$$\inf_{\pi \in \Delta(\Gamma(A) \times \mathbb{R}_+^N)} P(N) \sum_k \pi(b^k) \sum_{i \in \mathcal{I}} b_i^k$$

subject to the constraint that $a = (a_i)_{i \in \mathcal{I}} = (1, \dots, 1)$ is the unique subgame perfect equilibrium outcome of the game induced by π . To distinguish from the optimal (partial implementation) contract in the previous section, we call the solution to this (full implementation) problem the *u-optimal* sequentially mediated contract.

The next result (Theorem 5.6) generalizes the second part of the example in Section 3.2 and establishes that the u -optimal mediated contract involves obfuscation and takes the form of a canonical sequentially mediated contract.

Theorem 5.6. *There exists a u -optimal contract that takes the form of a canonical sequentially mediated contract. It randomizes over N sequential-move games, $(\hat{\gamma}^i)_{i=1}^N$, with the following probabilities*

$$\pi_1 = \pi_2 = \dots = \pi_{N-1} = \frac{P(1) - P(0)}{P(N) - P(0)}, \quad \pi_N = 1 - \frac{(N-1)(P(1) - P(0))}{P(N) - P(0)}. \quad 5$$

The order of moves in game $\hat{\gamma}^i$ is:

$$\begin{aligned} & (1, N, N-1, \dots, 2) \quad \text{for } i = 1, \\ & (i, N, \dots, i+1, i-1, \dots, 1), \quad \text{for } i = 2, \dots, N-1, \\ & (N, N-1, \dots, 1), \quad \text{for } i = N. \end{aligned}$$

The corresponding bonuses are given by

$$\begin{aligned} b_i^i &= \frac{c_i}{P(1) - P(0)}, \quad \text{for } i = 1, \dots, N-1, \quad b_N^N = \frac{c_N}{\pi_N(P(N) - P(0))}, \\ b_j^i &= 0, \quad \text{for all } i \neq j. \end{aligned}$$

Furthermore, the value of the principal's problem is $P(N) \sum_{i \in \mathcal{I}} c_i / (P(N) - P(0))$.

Before proving this result, we highlight that, unlike the two-agent example in Section 3, the move orders need to be deliberately structured to create a specific, sequential elimination of dominated strategies when there are at least three agents. For instance, the contract in Theorem 5.6 makes agent 1 the last mover in most game realizations to simplify her decision first. After taking care of agent 1, the contract then make agent 2 either the last mover or the mover before agent 1 in most games, and so on.

⁵The nonnegativity of π_N follows from the strict convexity of P . Since the increments $P(k+1) - P(k)$ are increasing in k , we have $P(N) - P(0) > (N-1)(P(1) - P(0))$.

Proof. We show that there is a unique subgame perfect equilibrium in the game induced by the contract in the statement of the theorem, in which all agents work if none of their predecessors shirks and shirk otherwise. First, whenever an agent shirks, all of his successors observe it and learn the extensive-form game they are playing. Moreover, their corresponding bonuses are zero in this extensive-form game by construction. Thus, it is strictly optimal for them to shirk if at least one of their predecessors shirks.

Given that all agents shirk at their off-path information sets, next we consider their on-path incentives, namely, their optimal choices at the non-trivial information sets when they are uninformed. Fix any $\varepsilon > 0$, and let $\tilde{b}_i^i = b_i^i + \varepsilon$ for all i and $\tilde{b}_j^i = b_j^i$ for all $j \neq i$. We will prove that, in the normal-form representation of this extensive-form game with bonuses $(\tilde{b}_i^i)_{i=1}^N$, all agents working is the unique strategy profile that survives iterated elimination of strictly dominated strategies (IESDS).

Consider agent 1 first. When he shirks, his expected payoff is always

$$\pi_1 P(0) \tilde{b}_1^1 = \frac{P(0)c_1}{P(N) - P(0)} + \pi_1 P(0)\varepsilon.$$

If all other agents work, agent 1's expected payoff from working is

$$\pi_1 P(N) \tilde{b}_1^1 - c_1 = \frac{P(N)c_1}{P(N) - P(0)} + \pi_1 P(N)\varepsilon - c_1 = \frac{P(0)c_1}{P(N) - P(0)} + \pi_1 P(N)\varepsilon,$$

which is strictly larger than his payoff from shirking. If a nonempty subset $S \subset \mathcal{I} \setminus \{i\}$ of other agents shirk, agent 1's expected payoff from working now is

$$\pi_1 [P(N - |S|) \tilde{b}_1^1 - c_1] = \frac{P(N - |S|)c_1}{P(N) - P(0)} + \pi_1 P(N - |S|)\varepsilon,$$

which is again strictly larger than his payoff from shirking. Note that by construction agent 1 is the last mover in all extensive-form games $\hat{\gamma}^i$ where $i \neq 1$. Thus, in any $\hat{\gamma}^i$ with $i \neq 1$, he is always informed before choosing his action whenever another agent shirks; consequently, he only occurs the effort cost in $\hat{\gamma}^1$. Hence, shirking is strictly dominated by working for agent 1.

Next consider agent 2. As agent 1, his expected payoff from shirking is always

$$\pi_2 P(0) \tilde{b}_2^2 = \frac{P(0)c_2}{P(N) - P(0)} + \pi_2 P(0)\varepsilon.$$

Given that agent 1 works at his on-path information set, when a subset $S \subset \mathcal{I} \setminus \{1, 2\}$ of agents shirk, agent 2's expected payoff from working is

$$\pi_2 [P(N - |S|) \tilde{b}_2^2 - c_2] = \frac{P(N - |S|)c_2}{P(N) - P(0)} + \pi_2 P(N - |S|)\varepsilon,$$

which is strictly larger than his payoff from shirking. That is, shirking is strictly dominated by working for agent 2 once shirking by agent 1 is eliminated.⁶ The same logic applies to agents $i = 3, \dots, N - 1$: given that agents $1, \dots, i - 1$ work at their on-path information sets, shirking is then dominated by working for agent i .

Next, given that all other agents work at their on-path information sets, agent N 's expected payoff from shirking is

$$\pi_N P(0) \tilde{b}_N^N = \frac{P(0)c_N}{P(N) - P(0)} + \pi_N P(0)\varepsilon,$$

whereas his expected payoff from working is

$$\pi_N P(N) \tilde{b}_N^N - c_N = \frac{P(N)c_N}{P(N) - P(0)} + \pi_N P(N)\varepsilon.$$

Therefore, working is strictly better than shirking for agent N .

Finally, the expected cost of this mediated contract to the principal is

$$\sum_{i=1}^N P(N) \pi_i \tilde{b}_i^i = \frac{P(N) \sum_{i=1}^N c_i}{P(N) - P(0)} + P(N)\varepsilon.$$

Since $\varepsilon > 0$ can be arbitrarily small and $P(N) \sum_{i=1}^N c_i / [P(N) - P(0)]$ is the least possible cost to incentivize all agents to work as an equilibrium, this proves the last statement in the theorem. $\square$

⁶Note however that working does not strictly dominates shirking for agent 2 in the original game. This is because in the sequential-move game $\hat{\gamma}^i$ where $i \geq 3$, agent 1 moves after agent 2 and can shirk even if agent 2 works, in which case shirking is better than working for agent 2 under this mediated contract.

Remark 5.7. Somewhat unexpectedly, the uniqueness requirement does not impose any additional cost to the principal under sequentially mediated contracts. Even the form of the u -optimal contract in Theorem 5.6 is similar to that in Theorem 5.1, except that the randomization probabilities and bonuses differ, and more importantly, the orders of moves in each sequential-move game now need to be chosen specifically, whereas there is more freedom under partial implementation.

Remark 5.8. The u -optimal contract is not unique. In fact, for any permutation of the set of all agents, one can construct a u -optimal contract following the logic of the contract in Theorem 5.6 in which the order of elimination in the IESDS procedure is determined by the permutation.⁷ Compared with the optimal public contract with a sequential-move game in Winter (2006) where agents are ordered according to their indices—those with higher costs move earlier—because of the convexity assumption of $P(\cdot)$ (i.e., effort complementarity), obfuscation is introduced to save the cost of incentivizing late movers under sequential mediation, which in turn makes the order of moves inconsequential.

Remark 5.9. Compared with the u -optimal static mediated contract in Halac et al. (2021), which randomizes over a large number of static games to introduce type uncertainty for agents and requires the elimination of strictly dominated strategies for all agent types, the u -optimal contract under sequential mediation randomizes over N sequential-move games and each agent is the first mover in one of these games. Furthermore, under sequential mediation N rounds of elimination of strictly dominated strategies—one round for each agent—are sufficient.

Remark 5.10. Note that the additional mileage gained from sequential mediation does not stem from the stronger solution concept of subgame perfect equilibrium instead of Nash equilibrium under static mediation. To see this, consider the example in Section 3.2. For either partial or full implementation, we can modify the

⁷The contract in Theorem 5.6 corresponds to the identity permutation.

game form in Figure 3 by letting Nature randomize over two additional sequential-move games, l' and r' , so that the previously off-path information sets are now reached with positive probability, as illustrated in Figure 5. Under this modified contract, the outcome in which all agents work can be *virtually* (i.e., with probability arbitrarily close to one) implemented as the Nash equilibrium outcome with essentially the same cost.⁸

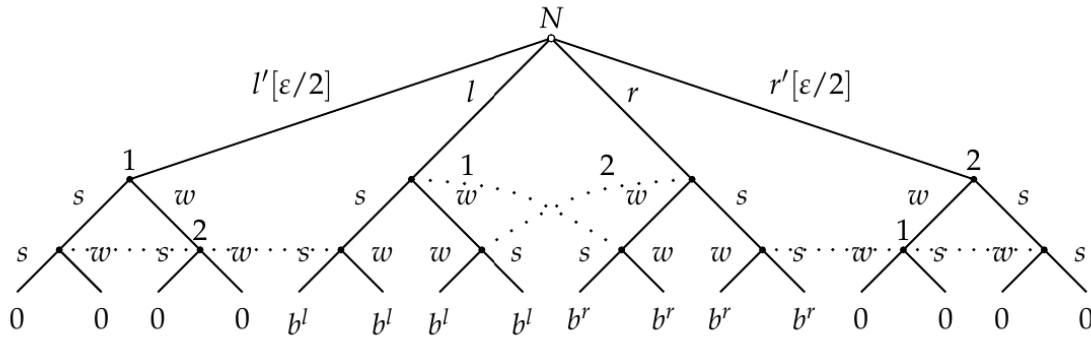


FIGURE 5. Virtual Implementation in Nash Equilibrium.

Finally, discrimination arises only superficially in the u -optimal contract in Theorem 5.6: although agents with the same effort cost are placed asymmetrically in the sequential-move games and receive different ex post bonuses, their expected bonuses are identical, as summarized in the following corollary.

Corollary 5.11. *The u -optimal sequentially mediated contract in Theorem 5.6 is non-discriminatory regarding agents' payoffs: if $c_i = c_j = c$ for agents i and $j \neq i$, then their expected bonuses are the same, which equal $P(N)c / (P(N) - P(0))$.*

6. CONCLUSION

This paper develops a framework of sequentially mediated contracts, in which a mediator designs not only transfers but also the timing and information structure

⁸The logic remains the same for the general case. It is an open question whether one can strengthen the result by removing the “virtual” term.

of agents' actions. By combining the informational flexibility of mediated contracting with the timing elements of sequential production processes, the framework shows how organizational structure itself can serve as an instrument of incentive design. Future work could restrict the set of feasible extensive-form games, explore other applications of dynamic contracting, or incorporate private information and communication.

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