Liquidity and Systemic Risk:
An Application to Stress Testing

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Abstract

The recent financial crisis re-ignites the concerns of systemic risk in the financial industry. It is the first time that systemic risk is caused by a lack of liquidity in the marketplace. To deal with this newly emerged systemic risk, researchers have been trying to develop various quantitative indicators. Despite of successful empirical results, these indicators are mainly empirically based and not capable of differentiating liquidity risk from other sources of risk such as market and credit. In this paper, we propose a parsimonious, option-based model to quantify liquidity and measure the magnitude of liquidity discounts. Then we apply the model to replicate the existing empirical indicators for systemic risk. We show that by incorporating our model, the existing empirical indicators can be more effective in measuring systemic risk. More importantly, because our model provides a consistent framework for existing empirical indicators, effective stress tests that overcome current common drawbacks can be devised.

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I. Introduction

The unprecedented financial crisis and the largest bankruptcy in U.S. history in 2008 prompted record awareness of systemic risk inside the financial industry. The rise of systemic risk is usually a result of a phenomenon one would expect in a bank run. Yet the 2008 crisis was not a conventional bank run but a liquidity vacuum. In other words, due to a lack of willingness to trade (buy), prices fall dramatically, and hence create a near-perfect correlation situation where diversification is impossible.

Systemic risk is not a new concept.[1] In "Systemic Risk: A Survey" by the European Central Bank in 2000, three types of systemic risk have been identified:

- bank run
- contagion
- failure in interbank systems

of which theories and empirical evidence are reviewed in details. However, the recent 2008 crisis defines a new systemic risk in our financial systems. Allen and Carletti (2013) view this new systemic risk as "banking crises due to asset price falls". They further define such a problem as "mispricing due to inefficient liquidity provision and limits to arbitrage."[2] Shin (2009) explicitly characterizes this liquidity-driven crisis as a new type of bank run. He contends that illiquidity, together with excess leverage and credit risk, ultimately affects nearly every financial institution. Also, Adrian and Shin (2010) document how financial institutions manage their leverage and the dynamics of the relationship between leverage and liquidity in the market.

Measuring this liquidity-induced systemic risk is an urgent topic. Shortly after the Lehman default, the Basel Committee has swiftly announced two liquidity ratios, LCR (liquidity coverage ratio) and NSFR (net stable funding ratio) to measure the liquidity

[1] This paper is authored by De Bandt and Hartmann (2000).
[2] Allen and Carletti (2013) define systemic risk as "it is the interactions of financial institutions and markets that determine the systemic risks that drive financial crises," and as a result, in addition to the three defined by the European Central Bank in footnote 1, they recognize a new kind of systemic risk that caused the recent crisis.

risk of a financial institution. However, researchers have critically criticized these two measures as being inappropriate and inaccurate.[3]

While Basel has been improving its liquidity ratios,[4] proposals have been provided by various regulatory bodies. Mainly they can be summarized as the following three areas. The first is to monitor correlations and price co-movements. Binici, Köksal and Orman (2013) study the Turkish banking system. They investigate the evolution of systemic risk in the Turkish banking sector over the past two decades using correlations of banks' stock returns as a systemic risk indicator. They report that the correlations between bank stock returns almost doubled in 2000s in comparison to 1990s, decreased somewhat after 2002 and increased again during the 2007-2009 financial crisis.[5] However, while equity correlations are certainly impacted by the crisis, it is not a clean measure of liquidity risk. In other words, there are too many causes for equity correlations to rise and hence the rise in equity correlations is not necessarily a signal for a lack of liquidity. In short, equity correlation is a necessary condition of the crisis, not a sufficient condition. We shall demonstrate that with our model for liquidity discounts, high equity correlations are a natural result of liquidity shortage. Furthermore, we shall demonstrate how to separate "normal" correlations from liquidity-induced correlations. Namely, with liquidity discounts, we can derive the correlations among illiquid asset values are the true indicator of systemic risk.

The second area is to observe default probabilities in the market. Combining default probabilities estimated with credit default swap (CDS) data with equity return correlations (as a proxy for asset correlations), Huang, Zhou, and Zhu (2009) create an indicator for systemic risk. They argue that their indicator for systemic risk is an ideal tool to measure a "distress insurance premium" – the theoretical price of insurance against financial distress. However, default probabilities estimated from CDS and equity correlations cannot be consistent with each other. Besides, neither measurement can gauge the magnitude of liquidity risk. Indeed, they are meant to measure credit and market risks. What we provide in this paper is to derive liquidity-implied asset

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[3] See, for example, Hong, Huang, and Wu (2014) for their empirical work and a comprehensive review provided. Bučková and Reuse (2011) provide evidence on the European banks.
[4] For example, see January 6, 2013 BIS report: "Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring tools".
[5] In addition, they explore possible determinants of systemic risk, the knowledge of which can be a useful input into effective macroprudential policymaking. Results show that determinants of systemic risk appear to be the market share of bank pairs, the amount of nonperforming loans, herding behavior of banks, and volatilities of macro variables including the exchange rate, U.S. T-bills, EMBI+, VIX, and MSCI emerging markets index.

correlations and default probabilities. As we shall demonstrate later, we differentiate liquidity and economic defaults and compute default probabilities and correlations consistenly. This then allows us to study separately how systemic risk is decomposed into credit and liquidity components.[6]

The third kind is to use expected shortfall (ES) or conditional Value at Risk (CoVaR). Both measures are based upon the popular market risk methodology – Value at Risk (VaR). Acharya, Pedersen, Philippon, and Richardson, (2010) use the expected shortfall as an indicator for systemic risk. They test 18 banks with their CDS data. They demonstrate empirically the ability of ES to predict emerging risks during the financial crisis of 2007-2009. De Nicolò and Lucchetta (2012) use ES to measure "real" liquidity risk with GDP growth and "financial" liquidity risk with the excess return of a portfolio of a set of specifically chosen financial firms. They use a vector-auto-regressive model to estimate and forecast the two variables and find significant out-of-sample forecasting power for tail real and financial risk realizations. They also argue that stress testing provides useful early warnings on the build-up of real and financial vulnerabilities.

A similar indicator, conditional Value at Risk (CoVaR), is proposed by Adrian and Brunnermeier (2011). A CoVaR is the VaR of the financial system conditional on institutions being under distress. They compute the contribution to systemic risk as the difference between the CoVaR conditional on the institution being under distress and the CoVaR in the median state of the institution. Using the universe of publicly traded financial institutions, they estimate CoVaR and study how firm characteristics such as leverage, size, and maturity mismatch can predict systemic risk contribution. They further argue that CoVaR could predict more than half of realized covariances during the financial crisis as early as 2006Q4.

One common deficiency shared by these papers is that they lack a theoretical liquidity discount model.[7] Without a liquidity discount model, one cannot measure systemic risk

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[6] Schwarz (2014) argues that widening interest rate spreads in the recent financial crisis could represent deteriorating asset liquidity and tries to separate market liquidity from credit risk so one can obtain a clean effect of liquidity risk. Her results reconfirm that market liquidity explains more than two-thirds of the widening of interest rate spreads. She concludes that the large role for market liquidity is due to the pricing of liquidity risk.

[7] The theory on systemic risk is voluminous. For example, Allen, Babus, and Carletti (2010) develop a model where institutions form connections through swaps of projects in order to diversify their individual risk. Upon the arrival of a signal about banks' future defaults, investors update their expectations of bank solvency. If their expectations are low, they do not roll over the debt and there is systemic risk in that all institutions are early liquidated. He and Krishnamurthy (2012) also derive a macroeconomic framework for quantifying systemic risk. Jobst (2012) proposes a systemic risk-adjusted liquidity model to study the

properly. For example the asset correlation implied by the Merton model is too low, as we shall show, to reflect the liquidity risk. As a result, it will lead to the under-estimation of systemic risk. Similarly, default probabilities and expected shortfalls measured without a liquidity discount model suffer the same deficiencies.

In this paper, we propose a parsimonic model for a liquidity discount. We show that this model is ideal to explain the systemic risk risen in the recent crisis. With our model, the three measures now commonly used in the literature are consistent with one another.

Our model for liquidity discounts leverages upon an option methodology that properly quantifies rapid asset falls due to the lack of liquidity. Via this model, we then show that large discounts in prices are not only possible but likely. We then examine the existing empirical literature that has been trying to provide a measure for this liquidity-included systemic risk. Finally, we propose an index that is similar to the liquidity gap highlighted by the Basel document.[8] We show that this index is superior to any existing indicator in measuring systemic risk and moreover predicting any liquidity crisis.[9]

Furthermore, as we shall explain in details later, our model can be used easily in conjunction with stress tests. Stress tests can be regarded as an effective way to reveal if a financial system is subject to systemic risk. A Financial Stability Review by the Bank of England (Bunn, Cunningham, and Drehmann (2005)) argues that because "stress tests can [...] help policymakers to gauge the potential implications of differing risks for the stability of the financial system as a whole" it can be regarded as an effective tool to detect systemic risk in the financial sector. And the article lays out the framework of the systemic stress test used by the Bank of England. An effective stress test can detect the vulnerability of the banking system and provide early warning signals. Ideally the stressed scenarios need to represent closely what could possibly happen in reality. Unfortunately criticisms have been raised that stressed scenarios are being unrealistic. The major problem with setting up stressed scenarios is two fold:

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impact of liquidity risk on banks. The proposed model is based on the Black and Scholes (1973) and Merton (1974) idea that the equity is a call on assets and the total liability is a default-free debt minus a put. He continues on to model the put option with the Black-Scholes formula. He gauges the general level of liquidity risk for a portfolio of institutions based on the current regulatory proposal aimed at limiting term structure transformation – the Net Stable Funding Ratio (NSFR), a measure for liquidity. His empirical tests use 212 banks globally.

[8] Basel Committee 2010 document entitled "Basel III: International framework for liquidity risk measurement, standards and monitoring".

[9] In a way, De Nicolò and Lucchetta (2012) propose a similar model and yet their model is econometrically based as opposed to option theoretically based.

- the models are not capable of linking losses to very fundamental economic factors
- hypothetical scenarios fail to consider interactions among various risk factors

We, by using the liquidity discount model, demonstrate that a single shock on the liquidity parameter (formal definition and details to be shown later) can generate simultaneous stressed risk factors such as high correlations, high volatility, high spreads, low P&Ls, low trading volumes, and even higher interest rates.

II. Model for Liquidity Discount

The 2008 crisis has been regarded as a liquidity crisis in the sense that prices were falling due to a lack of buyers. The liquidity of concern here, which is related to systemic risk, is the liquidity that results in simultaneous falls in asset prices. Therefore, this liquidity is not the commonly studied microstructure liquidity (although they share the basic economic reasoning) but a situation where demand of financial assets disappears (in this regard, we can view this liquidity as an extreme case of the typical microstructure liquidity). Figure 1 demonstrates in a linear manner how prices can fall with a lack of demand.

[Figure 1 Here]

In Figure 1, V represents the fundamental economy. As the economy improves, both demand and supply curves shift to the right. Yet, to explain liquidity discounts, the demand curve must move less than the supply curve.[10] For simplicity and without loss of generality, we let the demand curve stay the same. As a liquidity shock arrives in the market, the demand for the security disappears, resulting the demand curve to become completely vertical. In Figure 1 we assume that that market appetite is restricted at a level Q^*. As a result, the equilibrium price of the security decreases from P to P^* and the equilibrium quantity drops from Q to Q^*. This is a liquidity discount. Depending upon where the original equilibrium point is, the discount can be small (point A to point B) or big (point C to point D). We also note that the quantity is reduced as well. Of course, Figure 1 is an exaggerative (simplified) case; yet it represents the basic result of a liquidity squeeze of the market.

[10] To explain liquidity premiums, on the contrary, demand must move more than supply.

Continuing with analysis in Figure 1, we can derive the relationship of the liquid and illiquid prices. This is presented in Figure 2.

[Figure 2 Here]

The left panel of Figure 2 describes the relationship between the liquid price of the asset and the illiquid price of the same asset. As demonstrated in Figure 1, this relationship between liquid and illiquid prices is a kinked function. When the liquid price is high, there is no liquidity discount, as Figure 1 argues. When the price falls, the illiquid price starts to deviate from the liquid price. This results in the kinked function DBC in Figure 2 where ABC is 45-degree line where liquid and illiquid prices are equal.

Similarly we derive the result for quantity, which is presented in the right panel of Figure 2. In this figure, the illiquid quantity (vertical) is expressed as a function of the liquid quaintly (horizontal). Similar to the left panel, the straight line ABC represents a condition where there is no liquidity squeeze and the two quantities are equal by definition. If there is a liquidity squeeze at the point B, then the relationship becomes ABD.

The two panels of Figure 2 depict price discount and quantity discount as a concept of an option. On the price side, the discount mimics a put option and we can write $P^* = P - \xi$ where ξ is a put option on P^*. The strike price of the put option is clearly the value at point B. Similarly, on the quantity side, the discount mimics the call option: $Q^* = Q - \zeta$ where ζ is a call option on Q.

It is clear that the demonstration of Figure 1 and Figure 2 cannot be used without incorporating a proper model. In this paper, we first propose a "reduced-form model" where an exogenous put option is used and then an endogenous valuation is derived. Without loss of generality, we use the Black-Scholes-Margrabe model to model this put option:[11]

(1)
$$\begin{aligned} \xi &= K[1 - N(d_{S^*}^-)] - P^*[1 - N(d_{S^*}^+)] \\ \zeta &= QN(d_Q^+) - KN(d_Q^-) \end{aligned}$$

where

[11] The Black-Scholes model is constant strike and the Margrabe model (1978) is the Black-Scholes model with a random strike.

$$d_x^{\pm} = \frac{\ln x - \ln K + ½\sigma_*^2(T-t)}{\sigma_*\sqrt{T-t}}$$
$$\sigma_* = \sqrt{\sigma_x^2 + \sigma_K^2 - 2\rho\sigma_x\sigma_K}$$

where $x = P^*, Q$. We acknowledge that using this Black-Scholes-Margrabe model implicitly assume a log normal distribution of the illiquid stock price P^* (or quantity Q) and the strike price K which may or may not be appropriate. For example, when we use this model in conjunction with the corporate finance model to evaluate the value of assets of a bank (in the next section), we need the liquid asset price to follow the log normal distribution. Fortunately, with powerful computational capabilities, we can run numerical algorithms (such as binomial models or finite difference algorithms) to achieve consistency. These closed-form solutions facilitate our empirical work (that must deal with large amount of data) without altering the results qualitatively. The put and call analogies are not only intuitive but useful in modeling the 2008 crisis where liquidity squeeze impacted both price and volume, as we shall demonstrate in our empirical work.

III. Model for Asset Price – Liquid vs. Illiquid

As mentioned earlier, the recent crisis that prompted the unprecedented collapse of the banking system is due to the simultaneous falls of asset prices. These assets are owned by financial institutions in an interconnected manner (see Allen, Babus, and Carletti (2010)). As liquidity squeeze starts to surface in the market, banks are forced to sell assets in a very short period, causing prices to fall sharply, which in turn resulting in lenders (debt holders) to panic and start requesting additional collaterals. This phenomenon is known as a lack of funding liquidity. In other words, as the market values of these assets deviate drastically from their fair values, banks lose their ability to pay their short term debt obligations. Originally this could have been funded via short term borrowing. Yet, in a liquidity crisis, short term funding capacities dry up and liquidating assets becomes banks' only source of funding to repay their debts. As asset values drop rapidly, banks cannot liquidate their assets to generate enough cash to pay their debts. They then must file bankruptcy, leading to liquidity default.

This notion of default needs to be compared with the usual default that is caused by lack of profitability, which we term economic default.[12] When a firm cannot generate profits, it will then gradually lose its capital. When all of its capital is depleted, the firm then must file bankruptcy. In this situation, the firm also has lost its capability to raise any

[12] Wu and Hong (2012) term it insolvency.

equity as investors see no value of the firm. Formally, we define an economic default to be a default situation where a firm cannot raise any more capital, debt or equity. This occurs when the value of the assets falls below the total value of the debts (often called "negative equity value"). On the other hand, a liquidity default is a situation where a firm is profitable and yet lack of cash (or low liquidation value of assets) to pay for its immediate debt obligation.

Liquidity defaults have been monitored closely by accounting professionals. Annual auditing by CPA firms includes a going concern opinion as a result of the liquidity situation of a company. Liquidity here is defined as one year expected net cash flow. If there is not enough cash to pay expenses in the coming year, then the firm is deemed as illiquid and will not receive a favorable going concern audit. Note that the two new liquidity ratios by Basel III (LCR and NSFR) are quite similar to the liquidity ratios used in accounting.

In this section, we derive a model that explains this phenomenon. We use a multi-period version of the Merton model for determining banks' assets. We must use a multi-period model because there is no liquidity default in the Merton model. In a single period, all assets are liquidated at the end of the period and hence there is no differentiation of economic and liquidity defaults. In a multi-period setting, say two periods, we can differentiate if a bank defaults at time 1 due to liquidity (liquidity default) or due to profitability (economic default).

As in Merton (1974), Jobst (2012), Chen et. al. (2013), and many others, we model the equity of a bank by a call option. However, different from previous studies, we use the Geske compound option model (1979) as opposed to the Black-Scholes model in that our model of liquidity cannot be handled in a single period setting.

A. Economic Default (Liquid Value of Assets) – Merton/Geske Model

As in Merton (1974) and others, we define economic default as a company failing to raise equity and liquidity default as a company failing to make its immediate debt payment (interest or principal). Let the asset (only one class to begin with) of a financial company follow the Black-Scholes model:

(2) $$\frac{dA}{A} = \mu_A dt + \sigma_A dz ,$$

where z is Brownian motion and μ_A and σ_A are drift and diffusion, respectively. For simplicity, we assume that the firm has two debt payments (to be extended to multiple payments)[13] of K_1 and K_2 to be paid at T_1 and T_2, respectively.

When the economy is "normal" and the market is perfectly liquid,[14] it must be the case that defaults can occur only as the result of economic reasons. In Merton (1974) and Geske (1977) models, "economic default" is defined as $A_{T_1} < \bar{A}_{T_1}$, where $\bar{A}_{T_1}$ represents the total value of debts (see Geske for details). The equity value under this case, presented by Geske and Johnson (1984), is given by:

$$(3) \qquad E_t = A_t M\left(d^+_{A_t/\bar{A}_1}, d^+_{A_t/K_2}; \sqrt{\tfrac{T_1-t}{T_2-t}}\right) - e^{-r(T_1-t)}K_1 N_1(d^-_{A_t/\bar{A}_1}) - e^{-r(T_2-t)}K_2 M\left(d^-_{A_t/\bar{A}_1}, d^-_{A_t/K_2}; \sqrt{\tfrac{T_1-t}{T_2-t}}\right)$$

where $N(\cdot)$ is the uni-variate standard normal probability and $M(a,b;c)$ is the bi-variate standard normal probability with two limits a and b and the correlation c, and

$$d^{\pm}_{x/y_i} = \frac{\ln\frac{x}{y_i} + (r \pm \sigma_x^2)(T_i - t)}{\sigma_x\sqrt{T_i - t}}$$

We can also solve for the two debt values (Geske and Johnson (1984)) as follows:

$$(4) \qquad \begin{aligned} D_{t,T_1} &= A_t[1 - N(d^+_{A_t/K_1})] + e^{-r(T_1-t)}K_1 N(d^-_{A_t/K_1}) \\ D_{t,T_2} &= A_t\left[N(d^+_{A_t/K_1}) - M\left(d^+_{A_t/\bar{A}_1}, d^+_{A_t/K_2}; \sqrt{\tfrac{T_1-t}{T_2-t}}\right)\right] + e^{-r(T_1-t)}K_1\left[N(d^-_{A_t/\bar{A}_1}) - N(d^-_{A_t/K_1})\right] \\ &\quad + e^{-r(T_2-t)}K_2 M\left(d^-_{A_t/\bar{A}_1}, d^-_{A_t/K_2}; \sqrt{\tfrac{T_1-t}{T_2-t}}\right) \end{aligned}$$

with the spreads being:

$$(5) \qquad \begin{aligned} s_{t,T_2} &= -\frac{1}{T_2 - t}\ln D_{t,T_2} - r \\ s_{t,T_1} &= -\frac{1}{T_1 - t}\ln D_{t,T_1} - r \end{aligned}$$

Note that $N(d^-_{A_t/\bar{A}_1})$ and $M\left(d^-_{A_t/\bar{A}_1}, d^-_{A_t/K_2}; \sqrt{\tfrac{T_1-t}{T_2-t}}\right)$ are T_1 and T_2 risk-neutral survival probabilities respectively. The T_1 survival probability $N(d^-_{A_t/\bar{A}_1})$ is the probability of $A_{T_1} > \bar{A}_1$ and the T_2 survival probability $M\left(d^-_{A_t/\bar{A}_1}, d^-_{A_t/K_2}; \sqrt{\tfrac{T_1-t}{T_2-t}}\right)$ is the joint probability of $A_{T_1} > \bar{A}_1$ and $A_{T_2} > K_2$. The total risk-neutral default probability which is $1 - M\left(d^-_{A_t/\bar{A}_1}, d^-_{A_t/K_2}; \sqrt{\tfrac{T_1-t}{T_2-t}}\right)$ represents either default at T_1 or T_2. When the firm defaults at T_1, (i.e. $A_{T_1} < \bar{A}_1$), the firm is liquidated and A_{T_1} is the recovery value and split between

[13] But under a specific seniority order, as Geske and Johnson (1984) assume.
[14] The day-to-day usual and minor liquidity discounts are assumed away here.

the two debts. If the firm survives at T_1 but defaults at T_2 (i.e. $A_{T_1} > \bar{A}_1$ and $A_{T_2} < K_2$), then the first debt is paid in full and the recovery value for the second debt is A_{T_2}.

When the random recovery value A_{T_1} or A_{T_2} paid at T_1 or T_2 respectively should default occur combine with the default probabilities, it gives rise to the expected recovery value of $A_t\left[1 - M\left(d^+_{A_t/\bar{A}_1}, d^+_{A_t/K_2}; \sqrt{\frac{T_1-t}{T_2-t}}\right)\right]$ which is the current asset value multiplied by the default probability under the measure in which the random asset value is the numerarie.

B. Liquidity Default (Illiquid Value of Assets)

Accounting firms must issue going concern audits to public firms to reveal if these firms possess enough liquid assets to survive in the next year. Such audits do not consider economic consequences of the firms' investments but only examine the liquidity needs of these firms. Hence, for those firms that fail to acquire a favorable audit, they are subject to liquidity defaults.

Here, we can formally quantify (as opposed to human audits) such a liquidity default within the consistent framework that determines economic default, as described in the previous section. We define a liquidity default as:

(6) $$A^*_{T_i} < K_i$$

where $A^*_{T_i}$ is the liquidation value of total assets and K_i is the cash obligation due at time T_i. In other words, at any given time, as long as the liquidation value of the firm is insufficient to pay the cash flow due, the firm must default, regardless if the firm has large accrued profits. By definition $A^*_{T_i} \le A_{T_i}$ for all i. That is, the firm suffers some value when it sells its illiquid assets. This is reasonable in that when K_i is due and if the firm has insufficient cash to pay for it, then it must liquidate its illiquid assets and in such a circumstance the value is reduced.

In our two-period setting, when the economy is under liquidity stress, the asset value is compressed. As a result, the liquidity default is defined (going concern) as $A^*_{T_1} < K_1$. The equity value then is given by:

(3^*) $$E^*_t = e^{-r(T_1-t)}\mathbb{E}_t\left[E_{T_1}\left(\max\{A^*_{T_1} - K_1, 0\}, K_2\right)\right]$$

where $E_{T_1}\left(\max\{A^*_{T_1} - K_1, 0\}, K_2\right)$ is the equity value at time T_1, which is a call valuation with $\max\{A^*_{T_1} - K_1, 0\}$ as the underlying asset value and K_2 as the strike price.

Equation (3*) indicates that in a state where the firm survives, it must be that the firm has high enough asset value $A^*_{T_1}$ to pay for its current debt K_1. If so, the debt is paid for by the assets, and the firm's assets reduce to $A^*_{T_1} - K_1$. As a result, the Geske model will price the equity using $A^*_{T_1} - K_1$.[15]

Equation (3*) can be implemented only numerically. In the empirical work, $A^*_{T_1}$ is approximated as follows:

$$(7) \qquad A^*_{T_1} = A^*_t \exp\left(-½\sigma_A^2 + \sigma_A \int_t^{T_1} dz\right)$$

which is equivalent to the lognormal process without drift. Note that A^*_t is computed using Equation (1). The equity value at time T_1, $E^*_{T_1}$, is carried out using the Black-Scholes model by substituting $\max\{A^*_{T_1} - K_1, 0\}$ for the underlying asset value. To carry the call values at T_1 back to t to arrive at E^*_t, the standard binomial model with 100 steps is used.[16]

C. Bank's Balance-Sheet

We now combine the two models to evaluate the balancesheets of banks. While the literature of using the structural model of Geske (1977) to evaluate a firm's assets is voluminous, it has been mostly for non-financial firms. Financial firms are highly levered and hence the structural models generally fail. Recently, Chen et. al. (2014) argue that structural models are suitable for financial companies as long as we classify their assets and liabilities by their liquidity quality. Then they successfully estimate the default probabilities using only those illiquid assets and liabilities.

Banks borrow a disproportionably large amount of very short term liabilities (overnight) for their daily operations and borrow long term debts like non-financial firms. As a result, as Chen et. al. point out, a large (short term) portion of the leverage of a bank can be waived from calculating economic default probabilities. These large amounts of short term liabilities are "rolled over" to another almost identical short term liabilities. The ability to roll over short term liabilities is known as the funding liquidity.

[15] Given that this is only a one-period calculation, the last period of the Geske model is identical to the Black-Scholes model.
[16] Note that in the empirical work, the illiquid asset value is assumed to follow the same lognormal distribution as the liquid asset value.

When a bank is healthy, lenders of the short term liabilities comfortably collect interests and let the bank roll over. When the bank becomes risky, lenders can call off their lending and then the bank must liquidate some of its assets to pay for the liabilities. If the economy is strong and the market is liquid, then the bank can sell its assets at fair prices. If not, then the bank must suffer lower than fair prices when it sells its assets. A liquidity crisis is a situation where banks are losing their funding capability rapidly and all of them are selling assets at the same time. As a result, there is a direct connection between asset liquidity and funding liquidity.

Banks' balancesheets in general are very complex. Due to the high leverage nature of banks, regulations require banks carefully categorize their assets and liabilities in terms of liquidity. Chen et. al (2014) do a very detailed analysis of Lehman's balancesheet. In their Table 1, they demonstrate that overall Lehman has over 50% assets that are very liquid, over 40% assets that are semi-liquid, and 5% or so assets that are illiquid. In terms of liabilities, 50% of the liabilities are liquid, slightly above 25% of the liabilities are semi-liquid, and 20% or so of the liabilities are illiquid. They also survey the literature and conclude that using "net debt" is the most reasonable liability amount to estimate default probabilities. Here, without detailed bond issuance data from FactSet, we adopt the same methodology and use only data from long and short debts in CompuStat.

Similar to Chen et. al., we let the assets be classified as two classes (which can be generalized to m classes later): A_L and A_I where A_L represents the amount of liquid assets and A_I is the amount of illiquid assets. A_L (e.g. cash) is perfectly liquid and will not suffer from any liquidity discounts. On the contrary, the market value of A_I is A_I^*. Hence the market value of total assets (MVA) is $A_L + A_I^*$. Let the liabilities be also classified as two classes (n classes later) K_L and K_I where K_L is liquid like overnight funding and K_I is illiquid like bonds. The market value of debt (MVD) is $K_L + D_I$ where D_I is the market value of K_I considering liquidity discounts. Market value of equity (MVE) therefore is, by construction, $A_L + A_I - (K_L + D_I)$ in a liquid situation or $A_L + A_I^* - (K_L + D_I)$ in an illiquid situation. If $A_L = K_L$ which is the case of most banks, then MVE is $E = A_I - D_I$ or $E^* = A_I^* - D_I$. As a result, the equity value can be used to back out A_I using only K_I. Note that in our empirical work, a two-factor Geske is used and K_I consists of K_1 (one year debt) and K_2 (long term debt).

Wu and Hong (2012) have a similar recognition of the difference between the two types of default.[17] However, instead of a structural modeling approach, they use an econometrically estimated hazard rate for the risk of economic default and the TED spread[18] for the risk of liquidity default. Our approach differs from theirs in the following ways. First, we physically model default events as opposed to likelihoods of default. Second, our two default events are endogenously connected as opposed to separately measured. Third, because of the endogeneity, the two types of default allow us to calculate various existing measures for systemic risk.

IV Data

It is easier that we first explain the data used in this paper. In the next section when we start to introduce various measures of systemic risk we can then conveniently demonstrate the results.

We obtain constituent stocks in Russell 1000 financial sector index from January 1996 to December 2013, a total of 216 months. The data are collected from CRSP-CRSP/COMPUSTAT MERGED. There are five major inputs to the model. The first three are:

- K_1 (Debt in Current Liability): DLCQ from Fundamental Quarterly
- K_2 (Long-Term Debt): Total Debt (DLTTQ) minus Long-Term Debt Due in One Year (DD1Q)
- E (Market Capitalization): The Multiplication of Price-Close-Monthly (PRCCM) and Common Shares Outstanding (CSHOQ).

The fourth major input is the volatility of the equity, σ_E, which we compute using past 12 months of returns of market capitalization. The last major input is the convexity parameter of the liquidity discount model K which we calibrate to the model credit spread calculated by equation of equation (4) as $K = A(1 - 4(10\% - s))$.

Combining equation (3) and the volatility equation as follows:

$$
\begin{aligned}
\sigma_E &= \frac{A}{E}\frac{\partial E}{\partial A}\sigma_A \\
&= \frac{A}{E} M\left(d^+_{A_t/\bar{A}_1}, d^+_{A_t/K_2}; \sqrt{\tfrac{T_1 - t}{T_2 - t}}\right)\sigma_A
\end{aligned}
\tag{8}
$$

[17] They term economic default as insolvency.
[18] TED is an acronym formed from T-Bill and ED, the ticker symbol for the Eurodollar futures contract.

we can then solve for asset volatility σ_A and asset value A. Then we use equation (1) to solve for the illiquid asset value A^* by calibrating K to the credit spread of equation (4).

To see how the model works, we provide an example of Bank of America (BAC). Starting June of 2008, the equity volatility of BAC increased drastically, from 36% in June, 2008 to over 100% at the end of 2008. During this period, BAC experienced liquidity discounts in its asset values. Take December 2008 as an example. The market value of equity (MVE or E) is \$70.65 billion and the volatility σ_E is 100.1%. The book values of short and long term debts (K_1 and K_2) are \$364.65 billion and \$268.29 billion respectively. Assuming $T_1 = 1$ and $T_2 = 2$, we can then solve for asset value A and asset volatility σ_A to be \$682.45 billion and 30.28% respectively, using equations (3) and (8). Then we can use (1) to solve for the illiquid asset price A^* of \$ 535.945 billion using the asset volatility and a strike price K of \$502.199 billion. The strike price is calibrated to the credit spread via: $K = A(1 - 4(10\% - s))$ where the spread is calculated using equation (5) to be 3.3968%.

The sample size varies over time. The peak of the sample size (402 banks) is in December 2012 and the smallest sample size (189 banks) occurs in January 1997. The average size is 306 banks and the median is 308 banks. The banking classification information comes from Bloomberg. Then we classify the sample into 12 sectors in three broader categories as follows:

- Commercial
    - US commercial (305/129/229/236)
    - non-US commercial (90/42/64/63)
- Regional
    - eastern (71/35/55/59)
    - southern (108/35/74/75)
    - western (58/25/42/45)
    - central (70/34/56/59)
    - regional non-US (5/3/4/4)
    - super regional (10/7/7/8)
- Other
    - diversified (18/6/10/11)
    - fiduciary (7/4/5/5)
    - mortgage (2/1/1/2)
    - money center (1/1/1/1)

where the four numbers in each of the parenthesis represent max, min, mean, and median number of banks respectively.[19]

The summary statistics of the input variables are presented in Figure 3 (by time) and Table 1 (by sector). Figure 3 presents the cross-sectional average for each month of the input variables: volatility (VOL), short-term debt (STD), long-term debt (LTD) and market capitalization (MC). Due to the fact that the number of financial institutions in the sample varies over time, in Figure 3, we plot both total value and median value of each variable. Total value reflects the aggregate size of the market and median value presents an average concept. We note that the cross-sectional median of LTD gradually rose over time and peaked at the time of crisis and then declined afterwards (however, not so for the total value). Similar result is observed for the STD (both median and total values). MC and VOL move in opposite directions. Both clearly indicate the problem of the crisis. During the crisis, VOL reached record highs and MC dropped to pre-2000 levels. After the crisis, MC recovered and so did VOL.

[Figure 3 Here]

Table 1 presents results by sectors. Table 1 includes summary statistics of all four major input variables. A cross-sectional distribution is presented for each sector. From the difference between medians and means, it is obvious that the distributions of all four input variables (same can be said from observing 25%ile and 75%ile) are not normal. Also we find that standard deviations are all very large. The ratio of mean over standard deviation is far less than 2 in almost all variables and sectors. Hence, we conclude that these four variables have wide distributions, regardless of sectors.

In terms of size, the sector of Diversified Banks dominates all other sectors, followed by a distant second Super Regional. The median STD, LTD, and MC for the Diversified Banks are $237.07, $96.89, and $69.16 billion respectively and are only $5.37, $10.95, and $13.28 billion respectively for Super Regional. The volatility on the other hand does not present substantial differences across sectors. The levels of volatility range from 27% to 34%. We note that the volatility for the largest sector Diversified Banks is not the lowest, representing that size is not a risk factor for these banks.

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[19] Super regional bank contains Fifth Third Bancorp; Huntington; Wells Fargo; PNC Financial; KeyCorp; SunTrust Bank; Capital One, etc. Fiduciary bank includes Bank of New York Mellon Corp; Northern Trust Corp; and State Street Corp. Diversified banks has JP Morgan Chase & CO; Citi Group Inc; Bank of America Corp; Morgan Stanley; HSBC Holding Plc; Deutsche Bank AG; Dean Witter Discover & Co; Goldman Sachs Group Inc.

[Table 1 Here]

We use these inputs to compute liquid (using equations (3) and (8)) and illiquid (using equation (1)) asset values. The summary statistics of these values are presented in Table 2 with breakdowns by sectors. From the means and medians in Table 2, we do not find substantial differences among sectors over the entire sample period (1996 ~ 2013). This is because we do not differentiate the booming period and the crisis period. Later as we present time series results, it becomes clear that illiquid asset values deviate from liquid asset values by wide margins during the crisis period. We do observe, however, from Table 2 that the minimums of illiquid asset values across sectors are much lower than the minimums of liquid asset values. In fact, all except one sector (which is Fiduciary) have minimum values at 0. In other words, many banks show signs of liquidity default during our sample period. In the next section, we shall analyze more carefully these illiquid asset values. We discover that the values of illiquid assets carry crucial information of the liquidity health of the banks.

[Table 2 Here]

V Systemic Risk

The past crisis brings out a new systemic risk to which banks can be vulnerable and hence developing a proper measure for such a new systemic risk is an important task. Summarizing from what have been observed in the past crisis, three key variables have been used to measure such a new systemic risk. First, it has been observed that asset prices fall rapidly together causing correlations to rise to an unprecedented level. Second, a record number of bank defaults have occurred and default probabilities have worsened. Third, VaRs (Value at Risk) across banks have increased, and two measures, ES (expected shortfall) and CoVaR (conditional VaR), that are based upon VaR have shown strong explanatory powers of the crisis. In this section, we provide our results of these measures and compare them with the literature.

A. Correlation

In the classical finance theory, we often regard correlation in the context of market risk (e.g. CAPM), and as a result, mild levels of correlation is expected and observed. In the presence of the credit (default) risk, a new concept of correlation arises (known as the default correlation). However, the use of default correlation is narrow and applied to only CDOs which are commonly managed in SPVs and believed to be unrelated to the parent companies. Furthermore, the way this default correlation is modeled, known as Gaussian copula, is similar to the correlation under the market risk. As a result, this prevents the magnitudes of the correlation to be at extreme levels.

Using the put option analogy, we discover that correlations among asset values can easily reach extreme levels. This is intuitive as liquidity plays the role of a common factor that drives asset values. As liquidity becomes a primary concern, all assets correspond to it and hence correlations rise. Given that high correlation is regarded as a basic cause for systemic risk (hence bank runs), liquidity-induced high correlation must be evaluated for its severity during the 2007-8 crisis period.

From equation (1), we can then model any pair of financial assets, i and j, as $A_i^* = A_i - \xi_i$ and $A_j^* = A_j - \xi_j$. Then, the correlation between these two assets are defined as:

(9) $$\rho_{ij}^* = \frac{\mathrm{cov}[dA_i^*, dA_j^*]}{\sqrt{\mathrm{var}[dA_i^*]\,\mathrm{var}[dA_j^*]}}$$

where

$$\mathrm{cov}\left[dA_i^*, dA_j^*\right] = \sigma_i\sigma_j\rho_{ij}A_iA_j\left(1-\frac{\partial\xi_i}{\partial A_i}\right)\left(1-\frac{\partial\xi_j}{\partial A_j}\right)+\frac{\partial\xi_i}{\partial K}\frac{\partial\xi_j}{\partial K}\sigma_K^2K^2$$

$$\mathrm{var}\left[dA_i^*\right] = \sigma_i^2A_i^2\left(1-\frac{\partial\xi_i}{\partial A_i}\right)^2+\left(\frac{\partial\xi_i}{\partial K}\right)^2\sigma_K^2K^2$$

$$\rho_{ij} = \frac{\mathrm{cov}[dA_i, dA_j]}{\sqrt{\mathrm{var}[dA_i]\,\mathrm{var}[dA_j]}}$$

From equation (9), it is straightforward to verify that $\rho_{ij}^* \to 1$ as $K \to \infty$, $\sigma_K \to \infty$, or $\sigma_i, \sigma_j \to 0$; and conversely $\rho_{ij}^* \to \rho_{ij}$ as $K \to 0$, $\sigma_K \to 0$, or $\sigma_i, \sigma_j \to \infty$ This result is very intuitive. Recall that K is the "strike price" in equation (1), which represents the magnitude of the liquidity discount (as K measures the convexity of the liquidity payoff function). That is, the higher is K, the more severe is the discount. As both assets suffer liquidity discounts simultaneously, correlation under liquidity stress certainly increases and in the extreme case approaches unity. Conversely when K approaches 0, that is no "liquidity discount, then the "illiquid correlation" is just the same as the "liquid correlation". A similar conclusion can be drawn for σ_K. As the volatility of the liquidity discount factor increases, the "threat" of liquidity increases (that is, it pushes the put option value higher). Lastly, the impact from the individual volatilities, σ_i and σ_j, is opposite. This is also intuitive in that as individual volatilities rise, the impact of liquidity becomes relatively milder. In other words, as idiosyncratic risks become more dominant, common risk (i.e. liquidity) becomes less influential. This is analogous to the standard asset pricing model where the common risk is the market factor. As the idiosyncratic risks become more important, the impact of the market risk diminishes.

To visualize the results, we set up the following base case.

	Asset i	Asset j	K
Volatility σ_i, σ_j, σ_K	0.25	0.4	0.25
Partial $\partial\xi_i / \partial A_i$, $\partial\xi_i / \partial K$	0.45		0.45
Partial $\partial\xi_j / \partial A_j$, $\partial\xi_j / \partial K$		0.08	0.84
Price A_i, A_j, K	50	80	50

Figure 4 presents the conditional and unconditional correlations. The first panel of Figure 4 plots conditional correlation as a result of increasing liquidity squeeze (i.e. K). The second panel plots conditional correlation as a result of the volatilities of the two individual assets (i.e. σ_i and σ_j). For the sake of simplicity, we let them be equal. Lastly is the impact from the volatility of liquidity factor (i.e. σ_K).

In Figure 4, the perfectly liquidity correlation ρ_{ij} is set between –40% to +40% which is plotted on the x-axis. Various parameter values (i.e. K for convexity, σ_K for the fluctuation of the convexity measure, and σ_i for the volatility of the liquid asset return) are plotted on the y-axis. The liquidity-constrained correlation ρ_{ij}^* is plotted on the z-axis. As we can see, regardless what the value of ρ_{ij}, once the impact of lack of liquidity increases, the liquidity-constrained correlation ρ_{ij}^* can reach at +100%.

[Figure 4 Here]

We note that regardless of the unconditional correlation, liquidity drives the correlation to 100% when the effect of liquidity squeeze becomes strong. Binici, Köksal and Orman (2013) study the Turkish banking system and report that the equity correlations increased during the 2007-2009 financial crisis. Huang, Zhou and Zhu (2009) also study equity correlations but use them as proxies for asset correlations and avoid the trouble of extracting asset values from equity values. Different from Binici, Köksal and Orman and Huang, Zhou and Zhu, we do not study equity correlations but go directly to estimate asset values from equity values using the Geske model (1977) and study how asset correlations move over time, especially during the period of the crisis. Our advantage over Huang, Zhou and Zhu (2009) is that we can compare the correlations of illiquid asset values with those of liquid values. We find that equity correlations did rise during the period of the crisis; and yet the magnitudes are very mild compared the asset correlations, especially those of illiquid asset values.

Table 3 summarizes the correlation results of all the banks in the sample. The top panel of Table 3 reports the overall result in four different time buckets: 1996 ~ 2000, 2001 ~ 2007, 2008 ~ 2010, and 2011 ~ 2013. The first time bucket roughly describes the normal economic period. The second bucket represents the booming economy. The third bucket is the crisis. The fourth bucket is post-crisis. We see that overall the correlations among assets are 47.02% for liquid values and 44.61% for illiquid values. During the booming period, the "liquid correlation" is higher than the "illiquid correlation", 45.01% versus 44.20% respectively. Reversely during the crisis and post-crisis periods, illiquid correlations are higher than the liquid correlations. This is consistent with theoretical suggestion that liquidity stress will increase correlations.

[Table 3 Here]

We also observe that the liquid correlation during the crisis period is actually lower than the liquid correlation during the booming period. This also is consistent with the theoretical suggestion that liquid asset values rise together (as there are no liquidity concerns) which generate high correlations and do not necessarily fall together. It is the illiquid prices that fall together and hence illiquid correlations rise during the crisis period.

The second panel of Table 3 breaks the results down by sectors. For simplicity we only present booming and crisis periods. Same observation is made as in the upper panel that liquid correlations are higher during the booming period and illiquid correlations are higher during the crisis period. Comparing sectors, we find that regional banks are badly impacted liquidity and Fiduciary and Diversified banks are least impacted. US banks in general are worse than non-US banks.

B. Default Probability

Besides correlation, default probability is expected to detect systemic risk. Huang, Zhou and Zhu (2009) propose to use default probabilities, in conjunction with correlations, as a measure for systemic risk. Wu and Hong (2012) use conditional probability of default for the measure of systemic risk. Similar to our model, Wu and Hong also differentiate liquidity and economic defaults. Yet the two defaults are measured empirically separately while in our approach the two types of default are endogenously connected. In other words, our liquidity default probability is a better measure of systemic risk that emerged in the recent crisis.

As mentioned earlier, an economic default arises when a company does not generate enough profits to sustain its business. Losses gradually consume away equity and ultimately the firm runs out of capital and must default. Liquidity defaults are a completely different matter. A liquidity default is when a company, while profitable, is short of cash. In this situation, the company must liquidate its assets and the large discrepancies between the book/fair values of the assets and the market values of the assets can lead to bankruptcy of the company. In a previous section, we formulate the economic default condition as the total liquid asset value smaller than the total debt value $A_{T_1} < \sum_{i=1}^{n} D_i$ where D_i is the i th debt value (or the market value of cash flow K_i to be paid at time T_i) and the liquidity default condition as the total illiquid asset value smaller than the first cash flow due $A_{T_1}^* < K_1$. When the market is liquid, the two values are identical $A_{T_1}^* = A_{T_1}$ but when the market is not liquid, $A_{T_1}^* < A_{T_1}$. Hence, while $K_1 < \sum_{i}^{n} D_i$, the liquidity squeeze can be so severe that the company is economically solvent (i.e. $A_{T_1} > \sum_{i=1}^{n} D_i$) but liquidity insolvent (i.e. $A_{T_1}^* < K_1$). As we can imagine, under usual circumstances, economic default proceeds liquidity default, but in a severe situation liquidity default can proceed economic default.

In an example, we illustrate how a company's probabilities of default due to liquidity and economics differ. In a good economy, both default probabilities are small (near 0) and hence we observe no differences. However, when the profitability of the company deteriorates, the credit risk of the company increases, the economic default probability increases. If the company remains liquid (either the company has enough cash or there is no liquidity squeeze in the market), then the liquidity default probability is small. In this case, the difference is positive. As liquidity risk rises, the likelihood of default due to illiquidity rises. The default probability of liquidity becomes more dominant. Hence, the difference is negative.

Figure 5 takes the Bank of America (BAC) as an example. In the case of BAC, we see that prior to the crisis, the difference of economic default probability and liquidity default probability is positive, indicating that economic default is more likely liquidity default. However, the situations reverse during the crisis. Liquidity default becomes more likely and the difference becomes negative.

[Figure 5 Here]

Using the entire sample, we can see for the whole banking industry if liquidity default is more threatening than economic default during the period of crisis.

Table 4 summarizes the results of economic default probabilities versus liquidity default probabilities. We break the entire sample period into four time buckets: 1996-2000, 2001-2007, 2008-2010, and 2011-2013 to represent normal, booming, crisis, and post-crisis periods. The overall period shows that the liquidity default probability throughout the sample is roughly the same as the economic default probability, 1.12% versus 1.02% respectively. But the results of the four time buckets portrait a very different story. In both the normal and booming periods, economic defaults are more likely. In the normal period it is 0.98% (economic) versus 0.58% (liquidity) and in the booming period it is 0.73% (economic) versus 0.49% (liquidity). As expected, in the crisis period, the liquidity default probability is substantially higher than the economic default probability, 3.18% versus 2.45% respectively. It is interesting to observe that after the crisis is over, i.e. post crisis period, economic default becomes higher again (0.97% economic versus 0.57% liquidity).

When the results broken down to the sector level, we observe that Super-regional banks are the worst (10.72% liquidity versus 1.59% economic), followed by Regional-western (7.16% liquidity versus 3.52%). Although non-US commercial banks also experience in relative terms higher liquidity default probability (0.29%) than economic default probability (0.12%), the magnitudes are small. We also find that two sectors, Regional-central and Regional-eastern, did not suffer from liquidity crisis. Their liquidity default probabilities are smaller than the economic default probabilities during the period of crisis.

[Table 4 Here]

C. Expected Shortfall

Acharya, Pedersen, Philippon, and Richardson (2010) contend that systemic risk can be measured by expected shortfall (ES). They discover that ES increases with the institution's leverage and with its expected loss in the tail of the system's loss distribution. They show empirically the ability of ES to predict emerging risks during the recent financial crisis.

Expected shortfall is a VaR-based measure, defined as follows:

(10) $$\eta = \frac{1}{\alpha}\int_0^\alpha x_\gamma d\gamma$$

where x_γ is VaR (Value at Risk) at the probability γ. Formally, we define a VaR as:

(11) $$x_\gamma = \mu_A - p^{-1}(\gamma) \times \sigma_A$$

where $p^{-1}(\gamma)$ represents the critical value at the probability γ using the cumulative density function of $p(\bullet)$. In a Gaussian case, the density is a normal probability function and $p^{-1}(\gamma) = N^{-1}(\gamma)$. In our case, the probability density function for a bank's asset value is not necessarily normal.

Given that we have two asset values, liquid and illiquid, we then will have two VaR values, x_γ and x_γ^* for liquid distribution and illiquid distribution respectively. As a result, we can also compute:

(12) $$\eta^* = \frac{1}{\alpha}\int_0^\alpha x_\gamma^* d\gamma$$

We can then take a difference between the two of any given bank's asset, using equation (1):

(13) $$\begin{aligned}\eta^* - \eta &= \frac{1}{\alpha}\int_0^\alpha \left(x_\gamma^* - x_\gamma\right) d\gamma \\ &= \left(\mu_A^* - \mu_A\right) - \frac{\sigma_A^*}{\alpha}\int_0^\alpha (p^{*-1}(\gamma)(1+\Delta) - p^{-1}(\gamma))d\gamma \\ &= \left(\mu_A^* - \mu_A\right) - \frac{\sigma_A^*}{\alpha}\int_0^\alpha (p^{*-1}(\gamma)\Delta - \xi(\gamma))d\gamma\end{aligned}$$

which provides us an estimate of how much under-estimation of the expected shortfall without considering liquidity discounts. Note that $A = A^* + \xi$ and hence $\sigma_A = \sigma_A^*(1+\Delta)$ where Δ is the put option delta and is easy to compute.[20] The probability distribution for A can be derived by (via change of variable) the function $A = A^* + \xi$ and as a result $p^{-1}(\gamma) = p^{*-1}(\gamma) + \xi(\gamma)$, which results in the final line of equation (13). The integral in equation (13) is implemented numerically. The probability function of A^*, is assumed to be log normal with the volatility σ_A^* and hence $p^{*-1}(\gamma) = N^{-1}(\gamma)$.[21]

---

[20] In the Black-Scholes case, it is $-[1 - N(d_1)]$ and in this case $\sigma_A = \sigma_A^* N(d_1)$.

[21] There is slight discrepancy between the distribution assumptions of A and A^*. When using Geske (1977), we assume that A is lognormal. Yet when we evaluate liquidity discounts, we assume A^* is

Figure 6 demonstrates how we can compute expected shortfalls (ES) for liquid and illiquid assets. The first panel plots the liquid ES over time where median, the 25th percentile, and the 75th percentile are provided. As we can see that the median liquid ES rises over time and reaches its peak at the time of crisis, which is consistent with Acharya, Pedersen, Philippon, and Richardson (2010). Then it dissipates but still remains at an alert level till the end of the sample period. This can be compared with the results of the probability of default that remain non-trivial even after the crisis.

We also observe that the levels of ES are volatile after the crisis. While the general trend is going down, it fluctuates substantially. For example, after a considerable fall after the peak of the crisis, it rises again in early 2011. Another rise of ES is in early 2013. This indicates that the market overall is still not comfortable even though the government has implemented a series stimulus packages.

[Figure 6 Here]

The second panel of Figure 6 adds the additional ES due to liquidity. We can see from the figure that liquidity adds significantly to the existing ES. During the crisis period, it adds more than 10% to the existing ES. More importantly, the additional ES due to liquidity compensates the deficiency of the existing ES during the time of crisis. We note that from the end of 2008 till the end of 2009, the additional ES due to liquidity rises sharply as the existing ES falls. Hence, if we only use ES from the usual VaR result, then we severely underestimate the systemic risk in the industry.

VII. liquidity Gap as an Index for Systemic Risk

As we can see in the previous section, all three measures of systemic risk can be calculated with consistency under our model. As a result, we can construct a direct measure using the model itself. In other words, we propose a "liquidity gap" measure that is based upon estimated liquid and illiquid asset prices. The "gap" between the two values naturally represents the source of systemic risk. To come up with an intuitive measure, we take the ratio of the two values as follows:

(14) $\phi_{i,t} = A^*_{i,t} / A_{i,t}$

for the i th firm at time t.

lognormal. To resolve this inconsistency issue, we propose a full model and describe it in the Conclusion of the paper.

As we have seen in the previous section that those systemic risk measures are mainly empirical and furthermore cannot properly capture of the magnitudes of liquidity discounts. These measures, while useful, can only do half of the job. Our proposal for identifying systemic risk is a liquidity gap measure where a difference between liquid and illiquid asset prices are computed. Because of liquidity discounts, illiquid asset prices are always smaller than liquid asset prices, with the extreme cases where the two are equal. As a result, we can then compute the “liquidity gap” via a ratio of the illiquid asset price over the illiquid asset price.

There are several advantages of this new measure of systemic risk. First, it is consistent with the existing empirical measures. As we have shown in the previous section, while the existing measures successfully explain the crisis behavior, they are deficient in actually incorporating liquidity risk. What they have captured is mostly credit risk. The second advantage of our measure is that we can now decompose systemic risk into credit risk and liquidity risk. This is because in our model economic default and liquidity default are explicitly defined and consistently evaluated. Thirdly, our measure has a predicted power that is lacked from the existing empirical measures. As the asset prices are solved via a structural model, the equity information that reflects future expectations is then embedded in the asset values. Fourth, we use a multi-period structural model (i.e. Geske) as opposed to the single period Merton model. In doing so, we can then incorporate the entire debt structure of the bank into its risk. As indicated in Chen et. al. (2014), using the entire debt structure has an advantage of having the entire default probability curve of a bank and helps manage its risk more effectively. While in this paper, we only use two periods we can easily extend our analysis to multiple periods once proper data become available.

We calculate liquidity gaps for all the banks in the sample period monthly. We calculate cross-sectional market-value weighted average, equal-value weighted average, and median. We separate the commercial banks into different sub-indexes as stated above. We have the following several observations:

- Liquidity started to deteriorate at the third quarter of 2007. The threat remained till middle of 2011, for almost four years.
- The worst may have been over but the industry is still vulnerable.
- The most vulnerable sectors are Super Regional and Western.
- Larger banks are not necessarily more liquid.

Figure 7 presents three liquidity indices for the entire sample from January 1996 till December of 2013 which are (1) market value weighted, (2) equal weighted, and (3) median of the entire sample. We observe that all three indices deviate from 100% (i.e. showing liquidity discounts) in two periods: internet bubble burst in late 1990s and early 2000s and the Lehman crisis. During the Lehman crisis, all indices start to decline at the third quarter of 2007, six months earlier than Bear Sterns' default (March, 2008) and 12 months before Lehman's default (September, 2008). The indices also demonstrate that the discounts in the Lehman crisis were so much more severe than those in the internet bubble burst (post 2000).

[Figure 7 here]

Figure 8 through Figure 10 present liquidity indices for sub-sectors. Figure 8 divides the whole sample into US (top panel) and non-US commercial (bottom panel) banks. Banks in the US are more exposed to liquidity risk than non-US banks. We note that for the non-US banks, the equal-value-weighted measure and market-value-weighted measure are very close to each other, indicating that sizes do not matter. However, for the US banks, larger banks suffer more from the crisis than smaller banks, as the market-value-weighted index performs worst than the equal-value-weighted index.

[Figure 8 here]

Figure 9 presents the results for regional banks in five different regions – eastern, southern, western, central, and super regional. Out of the five regions, eastern, southern, and central present a similar pattern to national average. Super regional and western banks suffer the most during the Lehman crisis. The magnitude of their liquidity discount reached as low as 20%. Also super regional and western banks show no size effect, that is, the equal-value-weighted index and market-value-weighted index are close to each other. Different from super regional and western banks, other regional banks does show size effects. Southern and eastern banks demonstrate significant a size effect, both of which are in favor of small banks. For these two regions, the equal-value-weighted indices are substantially smaller than the market-value-weighted indices. Contrary to southern and eastern banks, central banks demonstrate that smaller banks are riskier, although not significant.

[Figure 9 here]

Figure 10 presents the last sub-sector of the banks that do not belong to any of the previous sub-sectors. These are fiduciary banks and diversified banks. Overall, this sector performs similar to the national average. We only note that the three measures (market-value-weight, equal-value-weight, and median) are close to one another, indicating no size effect in this sector.

[Figure 10 here]

V. Stress Test

It is a general consensus that stress tests can detect if an economy is vulnerable to systemic risk. Yet, such stress tests are hard to define. It's relatively easy to build bad stress tests. Basic errors in stress test construction can result from inappropriate manipulation of the data used in calculation of VaR. Simple operations on the everyday data can result in bad stress tests, too. Adverse market situations represent a different market "environment" than is reflected in the history of market price changes used in the VaR.

There are other issues. The first is that since stress tests are subjective, it is not possible to know whether all the relevant risks have been considered. The second is that since stress tests do not have an associated likelihood of occurrence (i.e., a probability), it's impossible to determine how to use stress results optimally.

According Schachter (undated), the goal of stress tests is to show sensible price and rate moves under such a stressed event if it were to happen in real markets. A good stress test should meet the following requirements: all price and rate movements (1) make *economic* sense when taken together; (2) are *relevant* to the particular positions and strategies represented in the portfolio; (3) are *relevant* to the current economic environment; and (4) have sufficient details to provide meaningful results on a position level basis.

Consistent with these requirements, the recent CCAR (Comprehensive Capital Analysis and Review) regulation defines the adverse scenario representing a moderate recession in the United States. This scenario includes level of real GDP declines by 2%; the unemployment rate rises to 9¾%; CPI rises to 4%; equity prices fall by 25%; the equity market volatility index jumps to 40%; house prices decline by 6%; and commercial real estate prices fall by 4½%.[22]

Financial institutions translate these descriptions into variables that are directly connected to their portfolio positions. According to Hull (2012), a set of testable scenarios are considered:

- Parallel shifting (up or down) 100-basis-point in a yield curve.
- Increasing or decreasing all the implied volatilities used for an asset by 20% of current values.
- Increasing or decreasing an equity index by 10%.
- Increasing or decreasing the exchange rate for a major currency by 6%.
- Increasing or decreasing the exchange rate for a minor currency by 20%.

Note that these scenarios are based upon market observables. There are several problems with such a design of stress test. First, the shocks of these market variables do not achieve the goal of stress test. At best they are extreme scenarios of the market risk. For example, a shock of 10% equity index is not a reflection of a stressed market. This is because the equity market can plummet by 10% but the economy can still be quite healthy, as long as other markets are not affected by this drop. Adding correlations between various markets can be helpful and yet the size of correlation is arbitrary.

Second, the models that are used to generate stressed losses are reduced-form models. Industry uses reduced-form models for their simplicity and computational efficiency to calibrate to market conditions (e.g. prices and volatility levels). However, this advantage can be the drawback for stress testing as the outputs of the models are quite predictable. In other words, it is quite easy to manipulate inputs to achieve the outputs desired.

Third, the translation of CCAR to the set of testable scenarios can be problematic. The fundamental reasoning for CCAR is to replicate past recessions and yet the set of testable scenarios may not be able to achieve that goal. This is because these testable scenarios are arbitrary shocks that may or may not represent a realistic recession situation.

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[22] CCAR also defines a severe recession with the unemployment rate at 12%; real GDP declines by 5%; equity prices fall by more than 50%; the equity market volatility index jumps above 70%; house prices decline by more than 20%; and commercial real estate prices fall by a similar amount. See "2013 Supervisory Scenarios for Annual Stress Tests Required under the Dodd-Frank Act Stress Testing Rules and the Capital Plan Rule" by the Federal Reserve Bank.

Furthermore, there is no causality between the shocks of the variables and the anticipated result of the economy; which defeats the very purpose of stress test in the first place.

A real stress test should not be perturbing the market observables but fundamental economic variables as CCAR lays out. Here we demonstrate how we can perturb market liquidity and investigate how firms will suffer from their losses in investments. From equation (1), we have $\sigma_A = \sigma_A^*(1+\Delta)$. And hence the sensitivity of the volatility with respect to the severity parameter is:

$$\begin{aligned} \frac{\partial \sigma_A}{\partial K} &= \sigma_A^* \left(\frac{\partial \Delta}{\partial K} \right) \\ &= \frac{n(d_{A^*}^-)}{A^* \sqrt{T-t}} \end{aligned} \tag{15}$$

and the spread sensitivity is proxied by the default probability sensitive as follows:

$$\frac{\partial s^*}{\partial K} \approx \frac{\partial p^*}{\partial K} \tag{16}$$

where $p^* = \Pr[A_{T_1}^* < K_1]$. There is no closed-form solution to equation (16) but we can compute the value numerically rapidly.

We choose 20 banks in our sample for the stress test exercise. We collect Morningstar ratings (available online)[23] for these banks and some selected CDS spreads (available online by MarkIt).[24] These results are given in Table 5.

[Table 5 Here]

It is clear to us that SLM Corp has the largest CDS spread, followed by Goldman Sachs (GS), Bank of America (BAC), AIG, and Citigroup (C). From our model, we also can compute the firm-wise implied spreads, which are given in the last column. We find that SLM has the highest spread (10.33%), followed by Bank of America (4.13%), Citigroup (3.06%) Genworth Financial (GNW, 2.10%). Goldman Sachs on the other hand, while ranked second highest in CDS spreads, enjoys a very low spread from our model.

In our stress test on liquidity, we shock the severity parameter K by 25% for each firm. As a result of more severe liquidity risk, credit spreads, asset volatility levels, liquid asset

[23] http://quicktake.morningstar.com/stocknet/bonds.aspx?symbol=gnw (or use any other ticker at the end).
[24] http://www.markit.com/cds/most_liquid/markit_liquid.shtml where only liquidity CDS spreads are given (Data and pricing (5 year tenor only) as of 08Oct2013. The numbers for CDS Contracts and Notional are provided by DTCC on week ending 27Sep2013.)

values (as a result of new volatility levels), illiquid asset values (as a result of new volatility and severity levels) and finally equity values will all accordingly change. The stressed results for the 20 banks are reported in Table 6.

[Table 6 Here]

We can see that now the results of deteriorating market liquidity. Such a deterioration of market liquidity results in simultaneous decreases in firms' equity values, increases in spreads and levels of volatility, and decreases in asset values. In terms of asset value, SLM is the worst bank (loss of 21.82% of its asset value), followed by GNW, BAC and C. Yet in terms of equity value, after SLM (97.46% loss in equity value), BAC is next and then C and GNW. Spread widenings present a similar picture as in equity value.

In terms of volatility, we see that SLM has the worst volatility outcome. Its shocked volatility will increase by 132%, followed by GS which has an increase of volatility by 99%. The best companies are C (12%) and FITB (14%). We note that the ranking of the volatility impacts is not quite the same as the ranking of credit spread impacts, even though they are highly correlated.

VIII. Conclusion and Future Research

In this paper, we propose a theoretical framework where the systemic risk risen in the recent crisis can be analyzed in a consistent manner. The theoretical framework is built upon a liquidity discount model that can explain large price falls due to the arrival of a liquidity distress. We show that under this framework, current proposals of measuring systemic risk, namely default correlations (which are proxied by equty correlations or comovements), expected shortfalls, and default probabilities can be computed in a consistent manner. Consequently, we then propose a more direct measure of systemic risk. This direct measure can be interpreted as a proxy of the popular concept of liquidity gap.

In our liquidity discount model, although reduced-form in its current format, asset values of financial institutions can be derived. From Geske's corporate finance model (1977, a multi-period extension of Merton (1974)), we can derive from the equity value the liquid asset value. Then using our liquidity discount model, we can then derive the illiquid asset value using the credit spread information implied by the Geske model. If the market is liquid, then liquidity discounts derived by our model will be nil. On the other hand, if the market is illiquid as in the case of 2008-9, our model will demonstrate a large liquidity discount and illiquid prices can deviate largely from liquid prices. The difference (in a format of a ratio) is a liquidity gap in asset values.

Using the data of a large number of financial institutions over a period of 1996 through 2013 (with a novel treatment of banks' balancesheets suggested by Chen et. al (2014)), we confirm that our liquidity gap measure can truthfully capture the crisis. In fact, our liquidity measure starts to show warning signs 12 months before the actual crisis (which is often marked as the time of Lehman default). In other words, our liquidity gap measure has a predicted power that is important to regulators to monitor the liquidity-induced systemic risk.

Our empirical results support entirely the existing literature that correlations, expected shortfalls, and default probabilities are all highly connected with systemic risk. Yet as opposed to empirically compute these measures separately independently, our liquidity gap measure is directly derived from the balancesheets of banks and therefore provides the most relevant measure of the risk. This is important not only in understanding and managing systemic risk, but also in providing a good tool in performing stress tests. Current stress tests focus on shocking credit spreads and market volatility. As our empirical results show, such partial shocks can provide biased results. Spread and volatility changes essentially are results of liquidity shocks. With our model, we can directly shock liquidity parameters (mainly the convexity parameter K) and changes of credit spreads and volatility can be measured. In other words, we can endogenize stress test parameters with our model. Moreover, given the theoretical framework, we can forecast future liquidity gaps. While we have not supplied enough details for the sake of the length of the paper, it is quite straightforward to run such simulations. To do that, we simply further parametize the convexity parameter K with a reasonable stochastic process.

One drawback of our current put option approach is that we fail to differentiate liquidity shocks between macroeconomic and idiosyncratic reasons. In our current model, the two are mingled into one liquidity parameter K. It is, however, crucial to separate the two different sources of liquidity risk to reflect the reality better. We should note that it is the macro-drive liquidity risk that is a concern of the industry. As Allen, Babus, and Carletti (2010) put it, banks are interconnected and risks face by one bank can easily spill over to other banks (its counterparties). Models of such kind need to be incorporated in the model of liquidity so we can understand systemic risk more clearly. While our empirical results strongly support macroeconomic shock for the period of the recent crisis, the put option value is estimated one company at a time without explicitly incorporating a macroeconomic factor.

To differentiate macroeconomic and idiosyncratic liquidity shocks, we must endogenize the put option for the liquidity discount. As Figure 1 already depicts, the falls of an asset price are an equilibrium result of demand and supply, which in turn is a result of changing economy. To endogenize the put option, we can model the fundamental economy with state variables. Consequently, we believe that once we endogenize the put option, we can explain the crisis better and furthermore provide a more effective tool for the regulators to monitor and manage similar risk in the future.

V. References

Acharya, Viral V., Lasse H. Pedersen, Thomas Philippon, and Matthew Richardson, 2010 (May), "Measuring Systemic Risk," working paper, New York University. MeasuringSystemicRisk.pdf

Adrian, Tobias and Hyun Song Shin, 2010 (April), "The Changing Nature of Financial Intermediation and the Financial Crisis of 2007-09," Staff Reports, Federal Reserve Bank of New York. sr439.pdf

Adrian, Tobias and Markus K. Brunnermeier, 2011 (October), "CoVaR," NBER. Adrian CoVaR.pdf

Allen, Franklin and Elena Carletti, 2013 (August), "What Is Systemic Risk?" Journal of Money, Credit and Banking, Supplement to Vol. 45, No. 1, pp. 121-7. jmcb12038.pdf

Allen, Franklin, Ana Babus, and Elena Carletti, 2010 (July), "Financial Connections and Systemic Risk," Working Paper 16177, NBER. w16177.pdf

Basel Committee document, 2010 (December), "Basel III: International framework for liquidity risk measurement, standards and monitoring".

Binici, Mahir, Bülent Köksal and Cüneyt Orman, 2013 (February), "Stock Return Comovement and Systemic Risk in the Turkish Banking System," working paper, Central Bank of Turkey. WP1302.pdf

Bučková, Veronika and Svend Reuse, 2011, "Basel III Global Liquidity Standards: Critical Discussion and Impact onto the European Banking Sector," Masaryk University.

Bunn, Philip, , Alastair Cunningham and Mathias Drehmann, 2005 (June), "Stress Testing as a Tool for Assessing Systemic Risks," Financial Stability Review, Bank of England. fsr18art8.pdf

Chen, Ren-Raw, N. K. Chidambaran, Michael Imerman, and Ben Sopranzetti, 2014 (August), "Financial Institutions in Crisis: Modeling the Endogeneity between Credit Risk and Capital Requirements," Journal of Banking and Finance, Vol. 45, p. 117-139,

Chen, Ren-Raw, William Filonuk, Dilip Patro, and An Yan, 2013 (Winter), "Valuing Financial Assets with Liquidity Discount: An Implication to Basel III," Journal of Fixed Income, Vol. 22, No. 3: pp. 45-63.

Cox, John, Stephen Ross, and Mark Rubinstein, "Option pricing: A simplified approach," Journal of Financial Economics, Volume 7, Issue 3, September 1979, Pages 229–263.

De Bandt, Olivier and Philipp Hartmann, 2000 (November), "Systemic Risk: A Survey," working paper No. 35, European Central Bank. ecbwp035.pdf

De Nicolò, Gianni and Marcella Lucchetta, 2012 (February), "Systemic Real and Financial Risks: Measurement, Forecasting, and Stress Testing," IMF Working Paper. wp1258.pdf

De Waala, Bernadine, Mark Petersena, Lungile Hlatshwayoa and Janine Mukuddem-Petersena, 2013, "A note on Basel III and liquidity," Applied Economics Letters, Vol. 20, No. 8, pp. 777-780

Federal Reserve Bank, 2013, "2013 Supervisory Scenarios for Annual Stress Tests Required under the Dodd-Frank Act Stress Testing Rules and the Capital Plan Rule."

Geske, Robert, 1977 (November), "The Valuation of Corporate Liabilities as Compound Options", Journal of Financial and Quantitative Analysis, Vol. 12, No. 4, pp. 541-552.

Geske, Robert, and Herbert E. Johnson, 1984 (June), "The Valuation of Corporate Liabilities as Compound Options: A Correction," Journal of Financial and Quantitative, Vol. 19, No. 2, pp 231-232.

He, Zhiguo and Arvind Krishnamurthy, 2012 (November 1), "A Macroeconomic Framework for Quantifying Systemic Risk," working paper, University of Chicago and Northwestern University. Arvind-Krishnamurthy-paper.pdf

Hong, Han, Jing-Zhi Huang, and Deming Wu, 2014 (September), "The Information Content of Basel III Liquidity Risk Measures," forthcoming, Journal of Financial Satiability. SSRN-id2344944.pdf

Huang, Xin, Hao Zhou and Haibin Zhu, 2009 (April), "A Framework for Assessing the Systemic Risk of Major Financial Institutions," Monetary and Economic Department Working Papers No 281, BIS. work281.pdf

Hull, John, 2003, Risk Management and Financial Institutions, 3rd edition, John Wiley & Sons.

Jobst, Andreas A., 2012 (August), "Measuring Systemic Risk-Adjusted Liquidity (SRL) - A Model Approach," IMF Working Paper. wp12209.pdf

Margrabe, William, 1978 (March), "The Value of an Option to Exchange One Asset for Another," Journal of Finance, Vol. 33, No. 1, pp. 177-186.

Merton, Robert C., "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates." Journal of Finance, Vol. 29, No. 2 (May 1974), pp. 449-470.

Pindyck, Robert S. and Julio J. Rotemberg, 1993 (November), "The Comovement of Stock Prices," Quarterly Journal of Economics, Vol. 108, No. 4, pp. 1073-1104. (2118460.pdf)

Schachter, Barry, undated, "Stress Testing for Fun and Profit," white paper, Caxton Corporation.

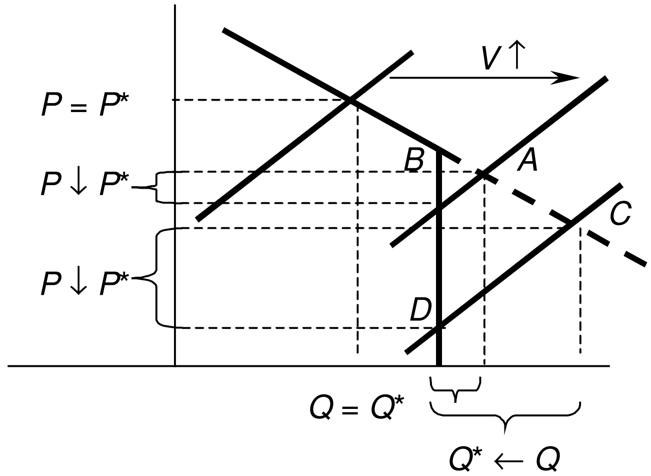
Schwarz, Krista, 2014 (May), "Mind the Gap: Disentangling Credit and Liquidity in Risk Spreads," Wharton School, University of Pennsylvania. spreads.pdf

Shin, Hyun Song, 2009 (Winter), "Reflections on Northern Rock: The Bank Run that Heralded the Global Financial Crisis," Journal of Economic Perspectives, Vol. 23, No. 1, pp. 101-119. jep%2E23%2E1%2E101.pdf

Wu, Deming and Han Hong, 2012 (November), "Liquidity Risk, Market Valuation, and Bank Failures," OCC and Stanford University.

shares outstanding http://ycharts.com/companies/BAC/shares_outstanding

Figure 1: Liquidity Discount



V represents the state of economy. As V goes up, both supply and demand curves move to the right. To explain liquidity discounts, demand must increase less than supply. For simplicity, we assume demand stays unchanged. The vertical demand describes the liquidity squeeze in an extreme case. Under liquidity squeeze, price falls ($P \rightarrow P^*$) and quantity falls ($Q \rightarrow Q^*$).

Figure 2: Liquidity as a Put Option

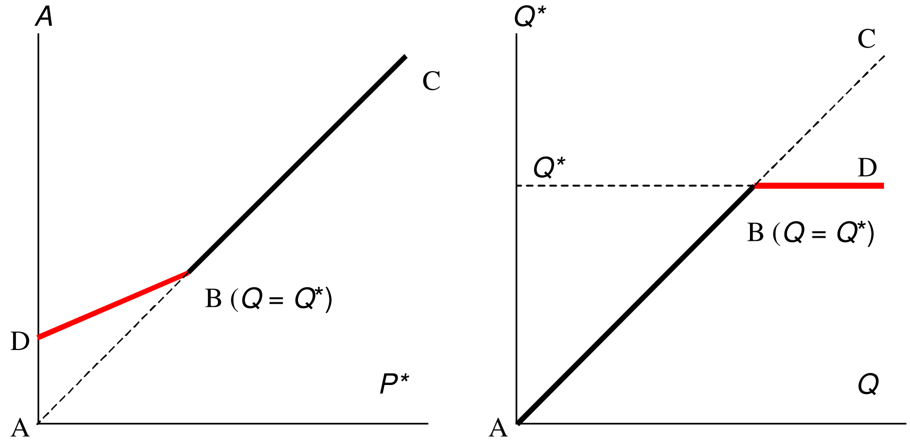
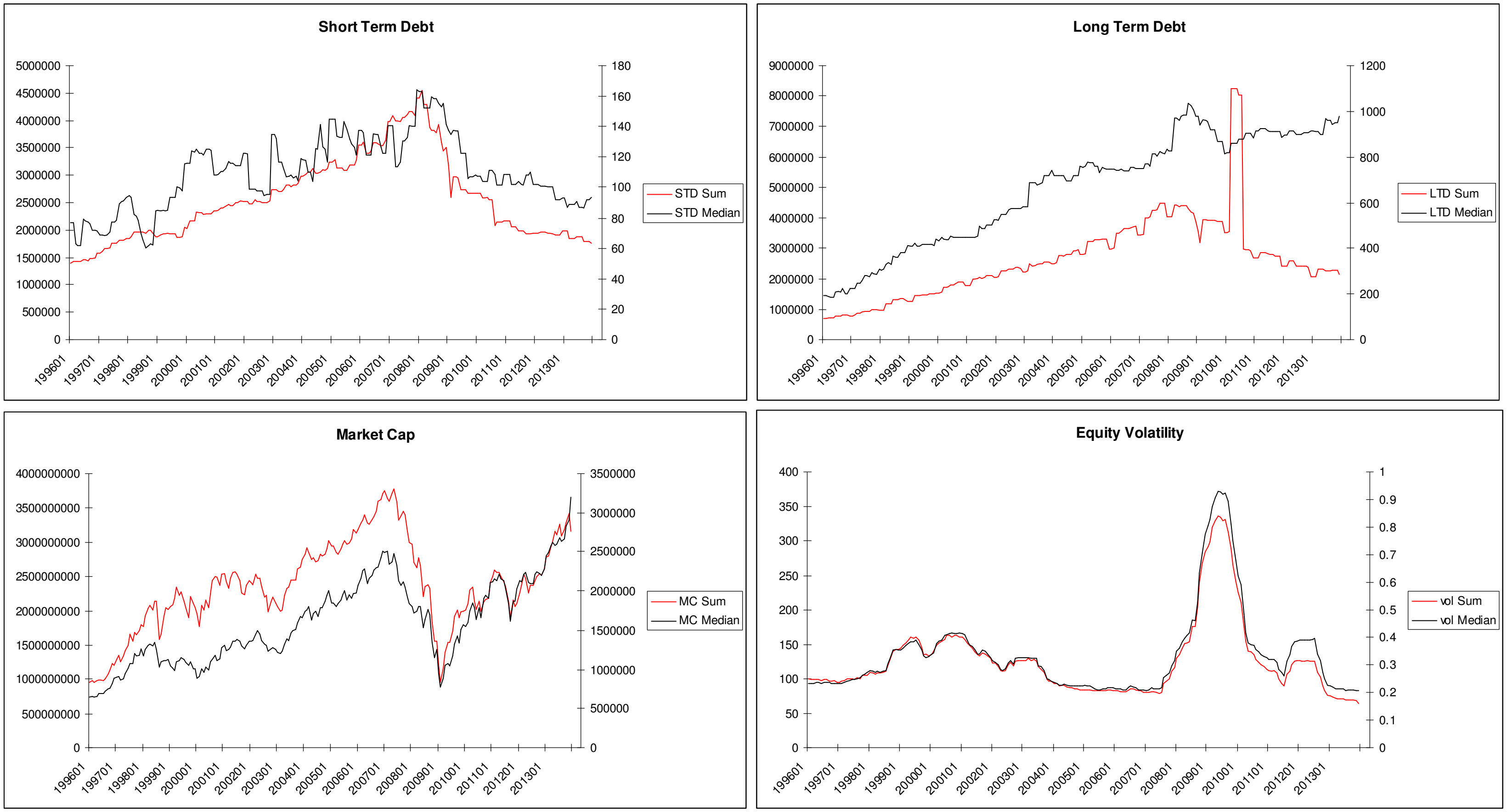


Figure 1 can be re-expressed as the relationship between liquid and illiquid prices (left panel) and quantities (right panel). Point B in Figure 1 separates liquid and illiquid states, above which is liquid and below which is illiquid. Hence on the left panel the relationship is a 45-degree line (where $P = P^*$) above point B and BD-line below point B. Similarly, on the right panel about point B it is BD-line and below point B is AB-line which is 45 degrees. As a result, the price discount is a short put and the quantity discount is a short call.

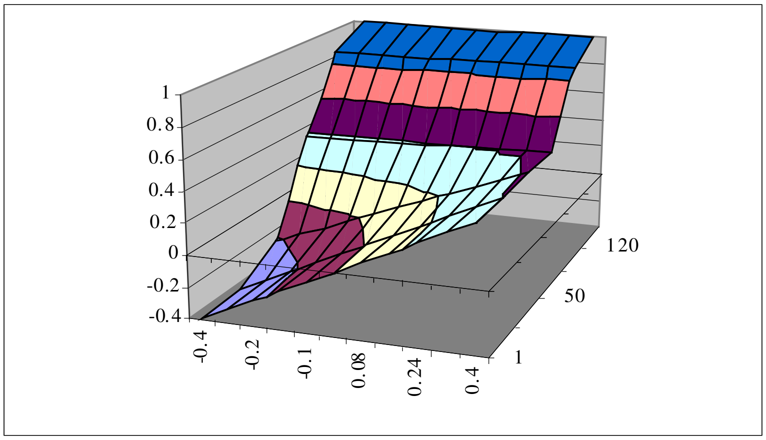
Figure 3: Time Series of Input Variables



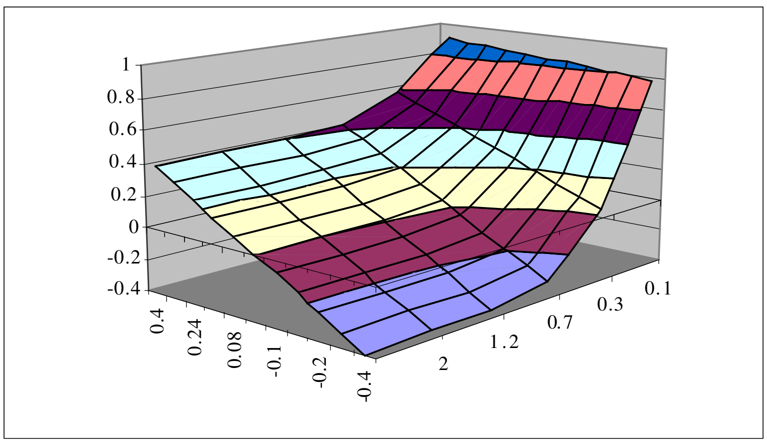
These are monthly series of input variables: short term debt, long term debt, market capitalization, and volatility. The black line is median which is labeled on the right vertical axis and the red line is total value which is labeled on the left axis.

Figure 4: Relationship between Conditional and Unconditional Correlations

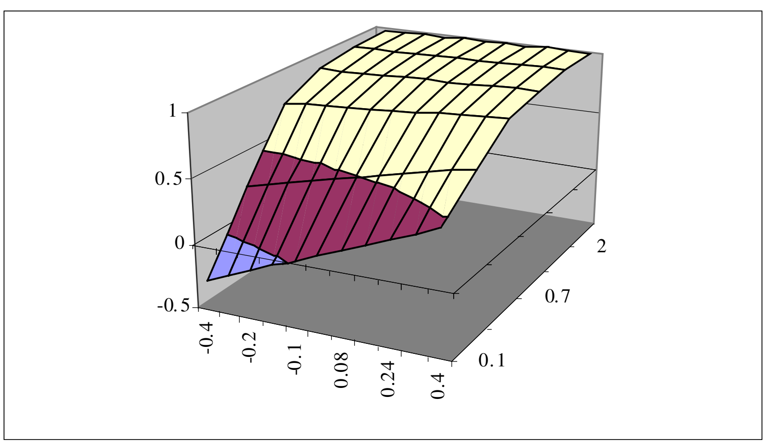
K from 0 ~ 120 (small to large)



σ_i from 0.1 ~ 2 (small to large)

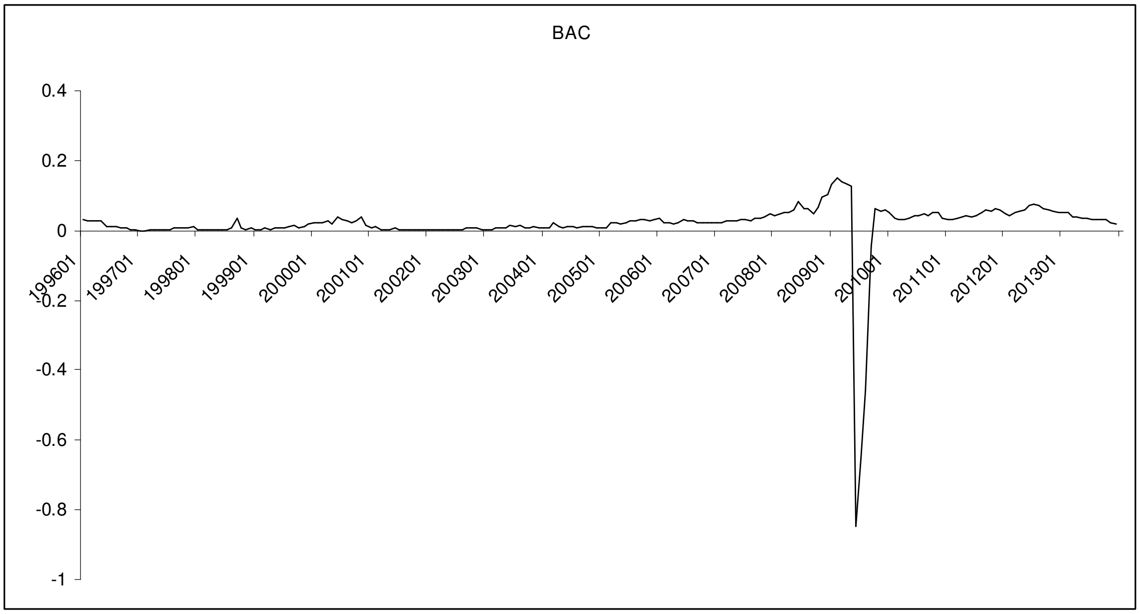


σ_K from 0.1 ~ 2 (small to large)



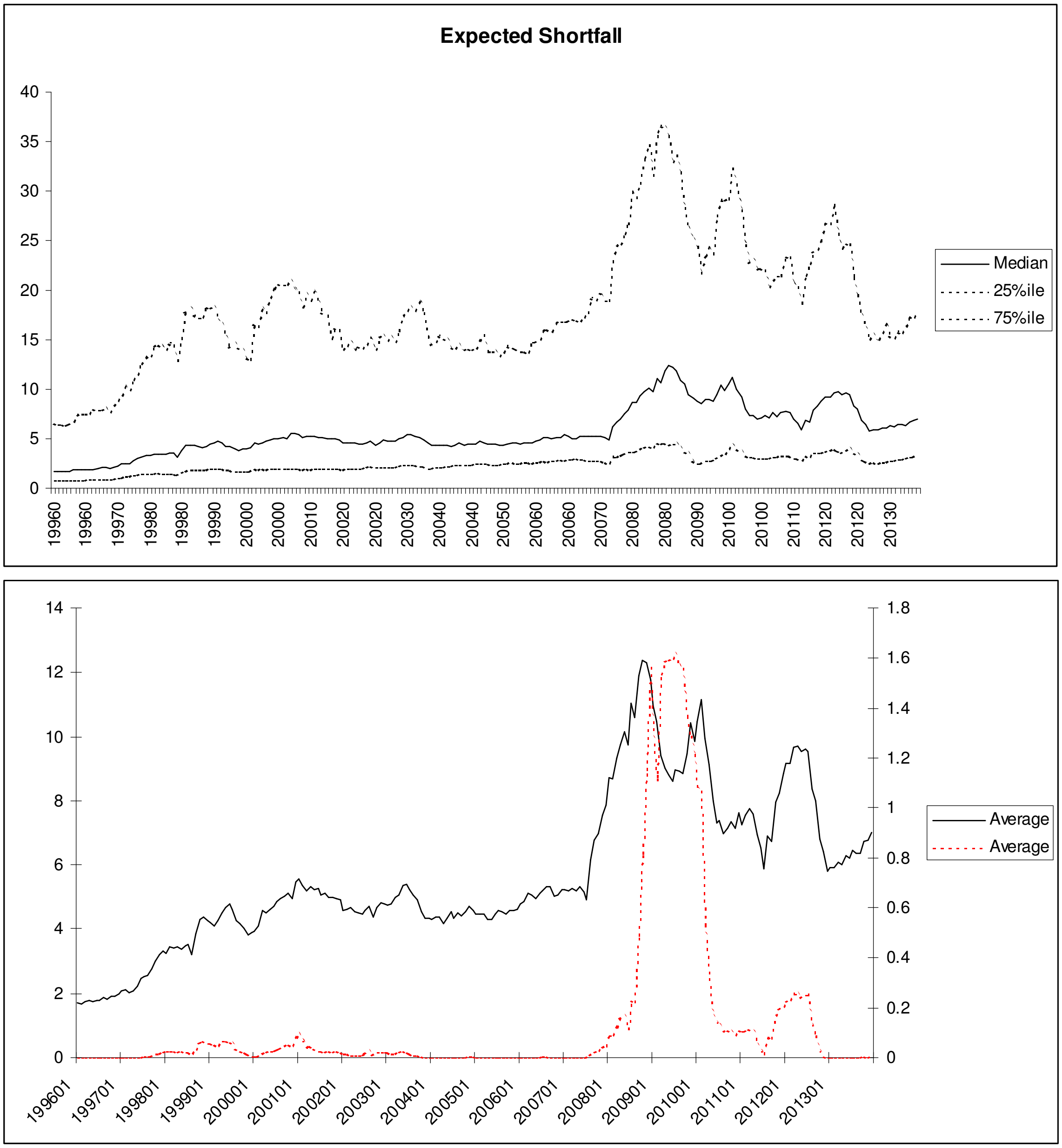
Note: The perfectly liquidity correlation ρ_{ij} is set between –40% to +40% which is plotted on the x-axis. Various parameter values (i.e. K for convexity, σ_K for the fluctuation of the convexity measure, and σ_i for the volatility of the liquid asset return) are plotted on the y-axis. The liquidity-constrained correlation ρ_{ij}^* is plotted on the z-axis.

Figure 5: Difference in Economic and Liquidity Default Probabilities (former minus latter)



This is an example of Bank of America (BAC). Before the first quarter of 2009, the economic default probability of BAC is higher than the default probability and the difference is positive. Afterwards, the liquidity default probability becomes higher and the difference becomes negative. After the fourth quarter of 2009, economic default probabilities are higher again. During the period where the liquidity probabilities are higher, BAC is more likely to default due to liquidity than to economics.

Figure 6 – Expected Shortfall



The upper panel plots the median, the 25th percentile, and the 75th percentile of the expected shortfall η based upon economic VaR x_γ. The lower panel repeats the median of the expected shortfall of x_γ but adds an extra expected shortfall due to liquidity $\eta^* - \eta$ where the former is computed based upon liquidity VaR x_γ^*.

Figure 7 – Liquidity Gap $\phi_{i,t} = A^{*}_{i,t} / A_{i,t}$: All Banks

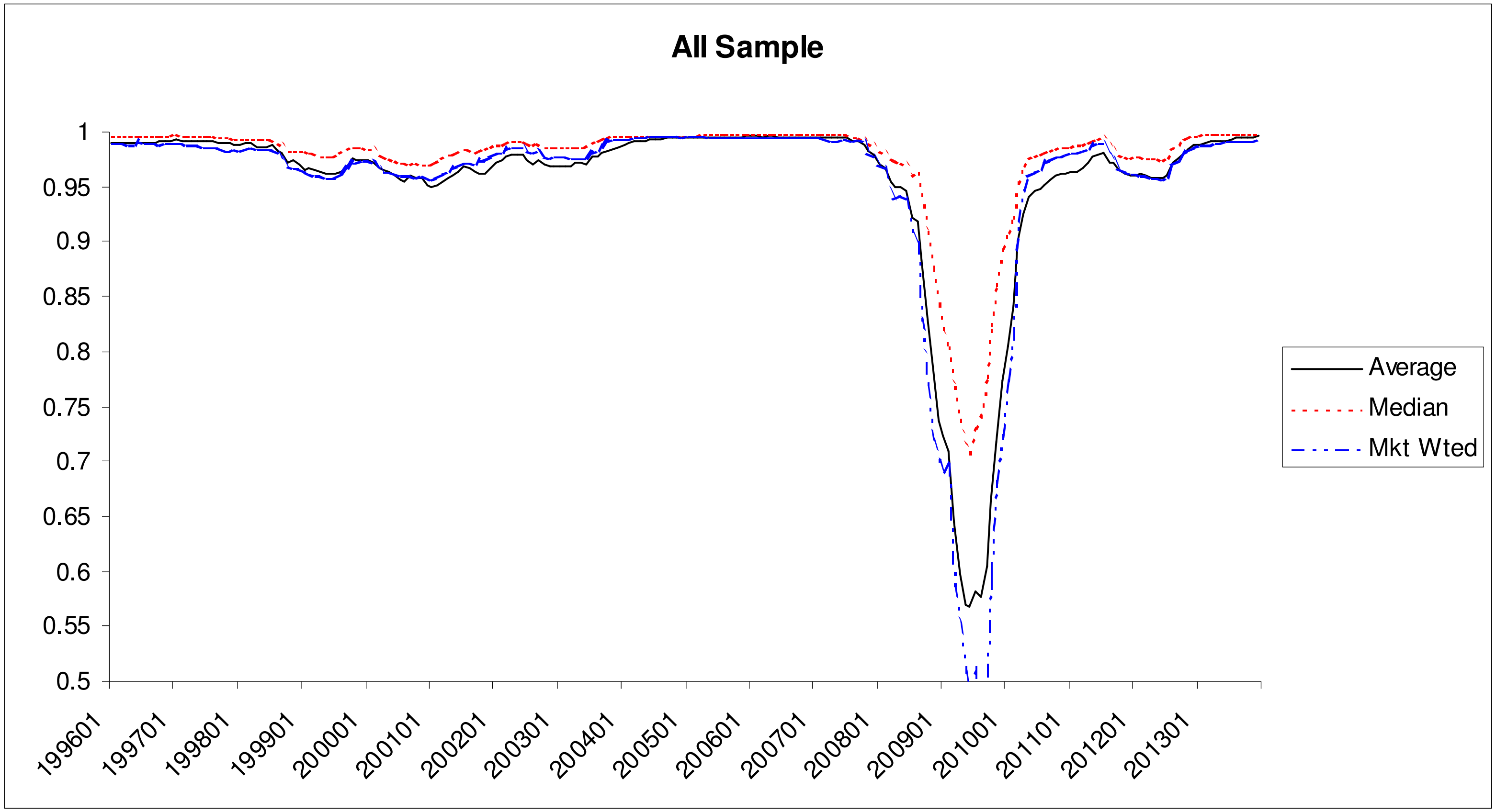
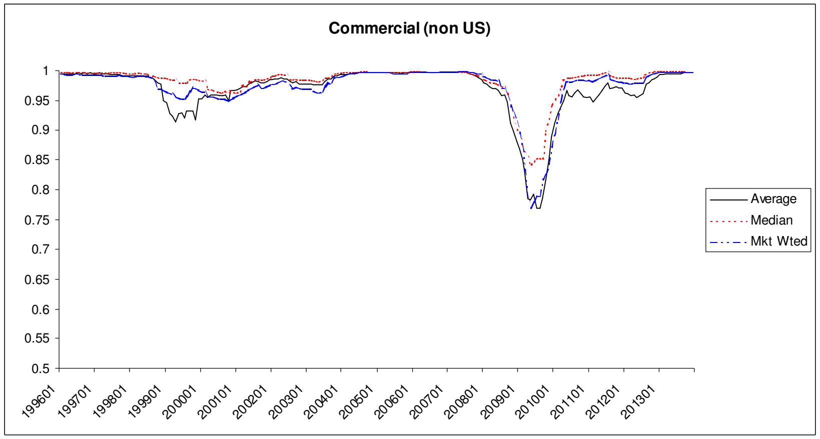


Figure 8 – Liquidity Gap $\phi_{i,t} = A^*_{i,t} / A_{i,t}$: Commercial Banks



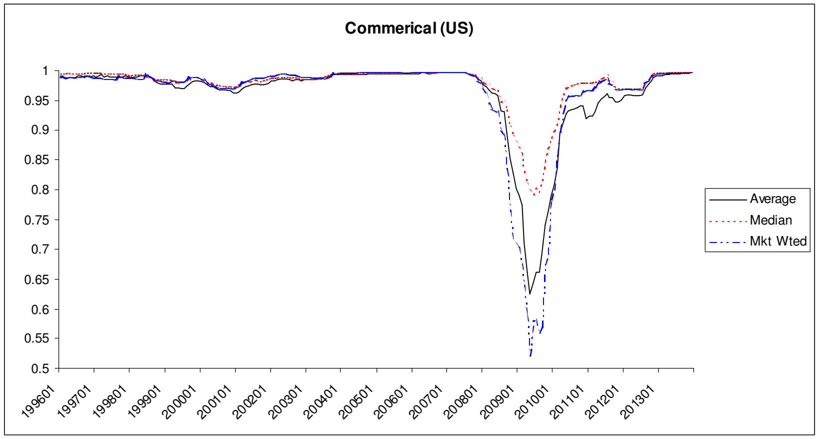
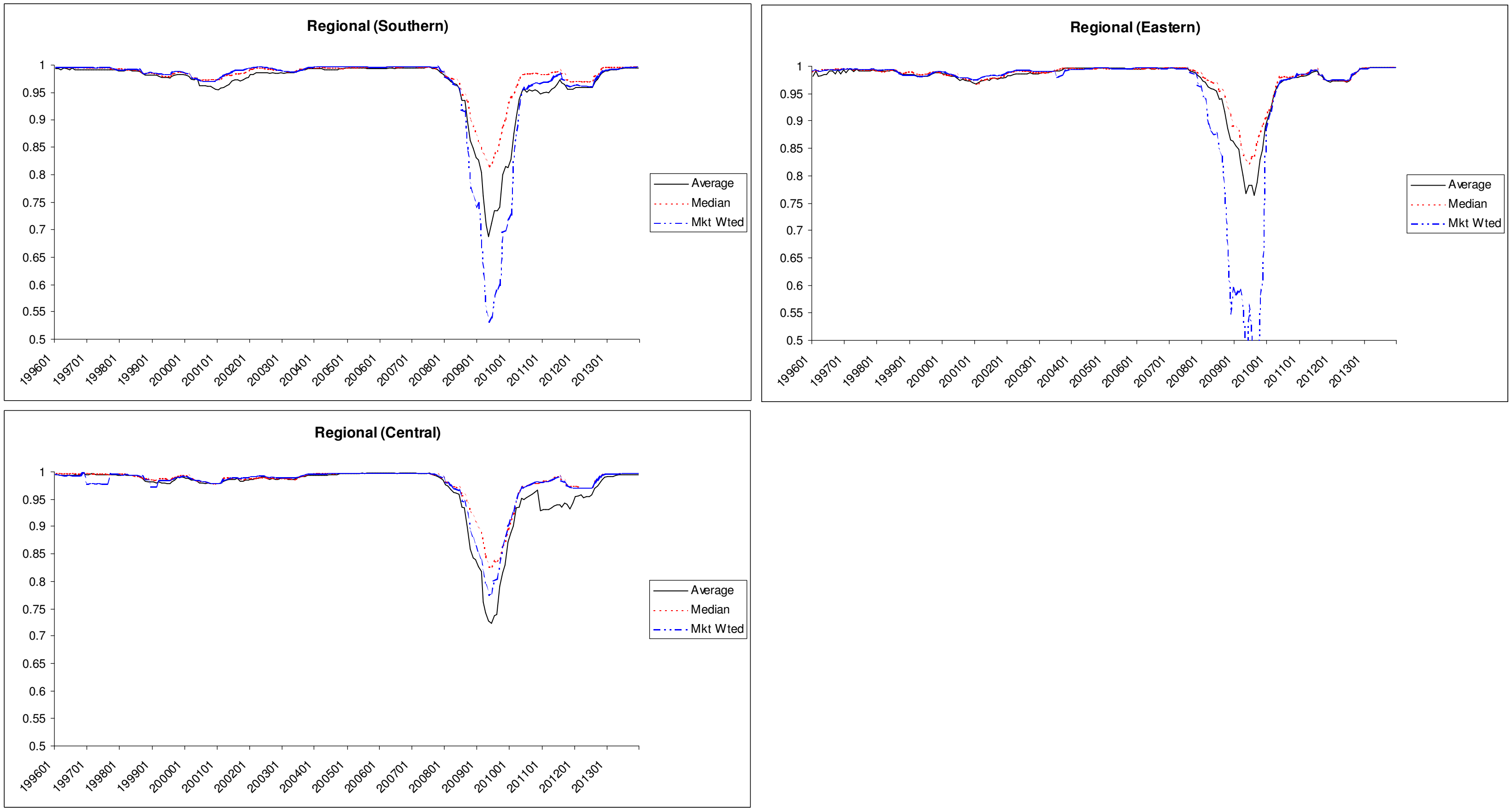


Figure 9 – Liquidity Gap $\phi_{i,t} = A^*_{i,t} / A_{i,t}$: Regional Banks



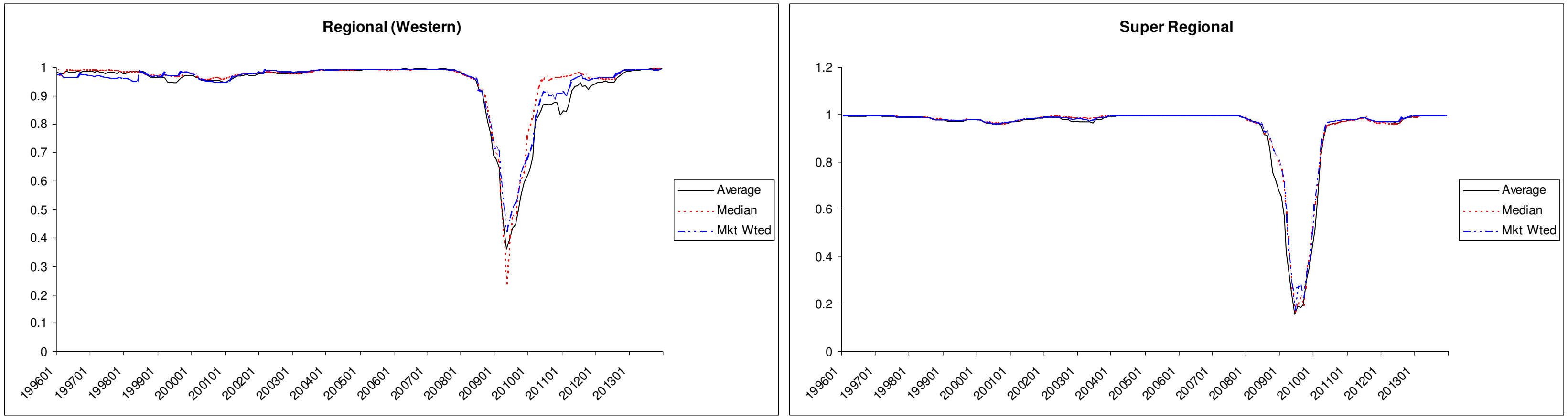


Figure 10 – Liquidity Gap $\phi_{i,t} = A^*_{i,t} / A_{i,t}$: Other Banks

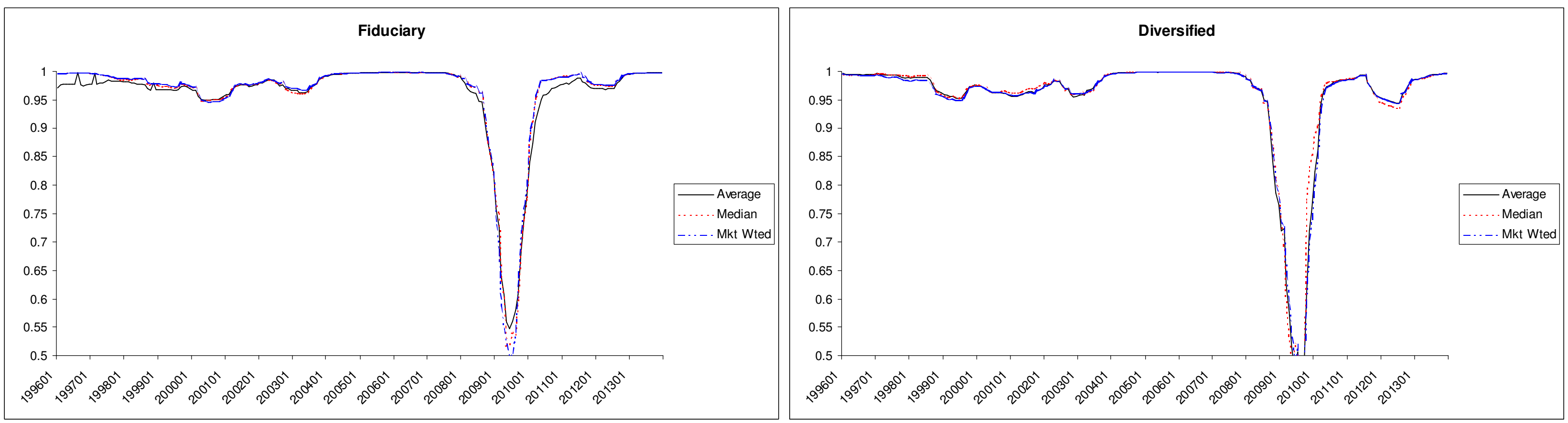


Table 1: Summary Statistics of Input Variables

Short Term Debt

	min	median	max	average	std.dev.	skew	kurt	#
Commer Banks-Central US	0	408	4,923	633	711	1.7874	3.6291	3,765
Commer Banks-Eastern US	0	308	19,936	790	1,683	6.0489	48.0615	3,244
Commer Banks Non-US	0	43	41,107	1,566	5,656	4.5607	20.7062	2,877
Commer Banks-Southern US	0	378	17,590	1,537	2,420	2.6377	9.2823	2,857
Commer Banks-Western US	0	70	8,326	381	782	4.0934	23.8898	4,080
Diversified Banking Inst	3,599	237,072	562,857	224,279	140,794	-0.0782	-1.0546	1,178
Fiduciary Banks	4	3,349	100,174	7,577	10,443	3.2365	20.6225	864
Super-Regional Banks-US	58	5,365	108,074	9,017	12,582	3.4523	15.2451	1,728
All	0	347	562,857	14,485	61,413	5.2151	27.6831	20,922

Long Term Debt

	min	median	max	average	std.dev.	skew	kurt	#
Commer Banks-Central US	0	233	4,859	530	770	2.7884	9.6033	3,765
Commer Banks-Eastern US	0	387	77,390	2,265	7,784	5.9441	38.7674	3,244
Commer Banks Non-US	0	850	45,946	3,133	7,639	3.8369	14.0980	2,877
Commer Banks-Southern US	0	264	23,380	1,955	4,386	3.1171	9.4343	2,857
Commer Banks-Western US	0	239	5,197	507	679	2.2456	5.8904	4,080
Diversified Banking Inst	3,905	96,894	558,830	131,107	119,363	1.1328	0.6839	1,178
Fiduciary Banks	29	3,842	20,372	4,851	5,062	1.4299	1.5511	864
Super-Regional Banks-US	278	10,953	267,158	20,455	33,591	4.0218	18.7625	1,728
All	0	489	558,830	10,548	42,556	6.6409	51.5228	20,922

Market Capitalization (,000)

	min	median	max	average	std.dev.	skew	kurt	#
Commer Banks-Central US	0	1,067	4,689	1,305	986	1.0007	0.4598	3,765
Commer Banks-Eastern US	0	815	15,161	1,663	2,478	3.0759	9.2504	3,244
Commer Banks Non-US	0	1,804	73,783	8,420	13,097	2.2981	5.3135	2,877
Commer Banks-Southern US	0	1,028	26,473	3,134	5,068	2.4890	5.7622	2,857
Commer Banks-Western US	0	708	9,392	1,104	1,248	2.5056	8.6490	4,080
Diversified Banking Inst	3,281	69,160	284,815	88,649	67,211	0.9058	-0.0746	1,178
Fiduciary Banks	-87	12,841	55,584	14,317	11,664	0.6514	0.0284	864
Super-Regional Banks-US	535	13,282	239,149	24,294	34,375	3.1505	10.6895	1,728
All	0	1,269	284,815	9,895	28,279	5.2148	31.4870	20,922

Equity Volatility

	min	median	max	average	std.dev.	skew	kurt	#
Commer Banks-Central US	0.0835	0.2969	1.7202	0.3491	0.2007	2.5691	9.0412	3,765
Commer Banks-Eastern US	0.1179	0.2968	2.3145	0.3465	0.1945	3.6264	23.5458	3,244
Commer Banks Non-US	0.0963	0.2708	1.9801	0.3325	0.2136	3.1490	14.6424	2,877
Commer Banks-Southern US	0.1290	0.3223	1.6681	0.3805	0.2107	2.2233	6.2546	2,857
Commer Banks-Western US	0.1289	0.3424	2.0502	0.4179	0.2621	2.4273	7.2733	4,080
Diversified Banking Inst	0.1202	0.3435	1.6212	0.3956	0.2506	2.5636	8.2051	1,178
Fiduciary Banks	0.1660	0.3450	1.3160	0.3883	0.2077	1.9546	4.4193	864
Super-Regional Banks-US	0.1122	0.2987	1.9078	0.3746	0.2812	2.6780	8.0472	1,728
All	0.0835	0.3105	2.3145	0.3707	0.2282	2.7264	10.3401	20,922

Table 2: Liquid and Illiquid Asset Values

Model Implied Liquid Asset Value (,000)

	min	median	max	average	std.dev.	skew	kurt	#
Commer Banks-Central US	76	1,941	10,714	2,459	1,878	1.1927	1.4006	3,746
Commer Banks-Eastern US	134	1,609	81,552	4,661	10,283	4.5109	22.5817	3,224
Commer Banks Non-US	33	3,696	139,099	13,106	24,650	3.2320	10.3650	2,859
Commer Banks-Southern US	22	2,035	51,487	6,574	10,847	2.4680	5.4616	2,841
Commer Banks-Western US	21	1,195	15,792	1,987	2,236	2.6894	9.8727	4,040
Diversified Banking Inst	11,016	424,653	1,274,252	436,944	277,589	0.1877	-0.8656	1,178
Fiduciary Banks	61	23,635	127,390	26,580	20,739	0.5864	0.4379	861
Super-Regional Banks-US	3,952	32,013	472,867	52,831	74,735	3.1934	10.2353	1,728
All	21	2,545	1,274,252	34,584	121,938	5.1250	27.9632	20,806

Model Implied Illiquid Asset Value (,000)

	min	median	max	average	std.dev.	skew	kurt	#
Commer Banks-Central US	0	1,930	10,715	2,444	1,877	1.1927	1.4250	3,746
Commer Banks-Eastern US	0	1,605	81,560	4,607	10,137	4.5558	23.2483	3,224
Commer Banks Non-US	0	3,599	138,675	12,989	24,559	3.2479	10.4742	2,859
Commer Banks-Southern US	0	2,009	51,476	6,468	10,753	2.5034	5.6578	2,841
Commer Banks-Western US	0	1,139	15,793	1,940	2,234	2.7153	10.0431	4,040
Diversified Banking Inst	0	419,454	1,274,342	432,446	276,825	0.2060	-0.8440	1,178
Fiduciary Banks	61	23,443	127,392	26,049	20,264	0.6194	0.6749	861
Super-Regional Banks-US	0	31,296	470,217	51,342	72,274	3.2111	10.6472	1,728
All	0	2,499	1,274,342	34,133	120,862	5.1591	28.4084	20,806

Table 3: Correlations

Level	All	96-00	01-07	08-10	08-13
Liquid	47.02%	43.92%	45.01%	41.49%	42.31%
Illiquid	44.61%	34.96%	44.20%	56.56%	56.17%

	All		01-07		08-10	
	Liquid	Illiquid	Liquid	Illiquid	Liquid	Illiquid
Commer Banks-Central US	30.01%	40.94%	46.57%	47.43%	18.66%	47.94%
Commer Banks-Eastern US	44.21%	53.32%	65.62%	66.12%	42.10%	65.57%
Commer Banks Non-US	57.26%	53.44%	61.59%	56.54%	57.62%	76.90%
Commer Banks-Southern US	28.30%	33.17%	62.91%	59.41%	23.56%	48.89%
Commer Banks-Western US	53.90%	60.24%	73.08%	74.73%	54.17%	65.05%
Diversified Banking Inst	66.86%	56.48%	75.07%	63.59%	86.62%	79.76%
Fiduciary Banks	62.82%	60.44%	63.53%	65.46%	73.56%	63.80%
Super-Regional Banks-US	18.39%	45.06%	33.70%	34.45%	29.92%	87.66%

Table 4: Default Probabilities (Economic versus Liquidity)

	Economic					Liquidity				
sector	All	1996-2000	2001-2007	2008-2010	2011-2013	all	1996-2000	2001-2007	2008-2010	2011-2013
All	1.02%	0.98%	0.73%	2.45%	0.97%	1.12%	0.58%	0.49%	3.18%	0.57%
Commer Banks-Central US	0.78%	0.59%	0.34%	1.96%	0.61%	0.33%	0.43%	0.18%	0.31%	0.33%
Commer Banks-Eastern US	0.30%	0.19%	0.12%	0.82%	0.08%	0.05%	0.10%	0.03%	0.13%	0.01%
Commer Banks Non-US	0.07%	0.05%	0.09%	0.12%	0.03%	0.13%	0.36%	0.33%	0.29%	0.00%
Commer Banks-Southern US	0.84%	0.98%	0.80%	1.49%	0.00%	0.59%	0.59%	0.41%	1.59%	0.19%
Commer Banks-Western US	0.78%	0.61%	0.21%	3.52%	0.06%	1.34%	0.33%	0.14%	7.16%	0.30%
Diversified Banking Inst	5.90%	5.31%	7.33%	10.97%	9.02%	2.57%	3.30%	3.45%	4.90%	1.93%
Fiduciary Banks	0.80%	0.79%	0.81%	1.49%	0.11%	0.85%	0.59%	0.62%	2.65%	0.02%
Super-Regional Banks-US	0.35%	0.19%	0.07%	1.59%	0.03%	1.79%	0.01%	0.00%	10.72%	0.00%

Table 5: Basic Information of the 23 banks

Name	Morning Star Rating	CDS	Model-implied Spread
AIG	BBB-	100/98.5	0.00%
ALL	BBB	100/44.5	0.00%
AXP	A-	100/47	0.00%
BAC	BBB	100/109	4.13%
BBT	A-		0.00%
BK	A		0.00%
BRK.A	High		0.00%
C	A-	100/97.5	3.06%
COF	A-		0.00%
FITB	A-		0.00%
GNW	BBB-		2.10%
GS	BBB+	100/128.5	0.01%
PFG	BBB		0.00%
PNC	A-		0.00%
PRU	BBB-	100/109.5	0.07%
SLM	Middle	500/320	10.33%
STI	BBB		0.03%
STT	A-		0.01%
TRV	A-	100/43.5	0.00%
USB	A+		0.00%

Table 6: Stress Test Results of the 23 Banks

Name	Drop in Model-implied Asset Value	Drop in Equity Value	Model-implied Spread Change	Model-implied Volatility Shock
AIG	1.02%	21.66%	1.50%	12%
ALL	1.02%	17.52%	1.50%	58%
AXP	1.02%	22.99%	1.50%	74%
BAC	3.62%	65.01%	6.04%	75%
BBT	1.02%	27.87%	1.50%	95%
BK	1.02%	26.03%	1.50%	74%
BRK.A	1.02%	16.71%	1.50%	59%
C	3.22%	59.25%	4.86%	12%
COF	1.02%	25.93%	1.50%	74%
FITB	1.02%	23.77%	1.50%	14%
GNW	3.88%	38.59%	3.81%	25%
GS	1.03%	26.08%	1.51%	99%
PFG	1.02%	18.40%	1.50%	12%
PNC	1.02%	28.32%	1.50%	74%
PRU	1.08%	28.95%	1.58%	18%
SLM	21.82%	97.46%	12.86%	132%
STI	1.04%	28.48%	1.53%	98%
STT	1.03%	29.68%	1.51%	88%
TRV	1.02%	16.56%	1.50%	60%
USB	1.02%	22.78%	1.50%	76%