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Optimal Phasing and Inventory Decisions for Large-Scale Residential Development Projects

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Abstract Land developers in select economic environments have been found to build in large increments and hold substantial amounts of inventory despite their ability to mitigate risk by phasing the production of residential lots. Such behavior was observed in numerous metropolitan areas throughout the southeastern and southwestern United States in the years leading up to financial crises, resulting in inventories of tens of thousands of lots in cities such as Atlanta, Las Vegas and Orlando, just to name a few. The model presented in this paper explores the rationale behind the choices made by developers in these markets and others by extending the real options framework to concurrently estimate optimal phasing and inventory decisions for large-scale residential development projects. Modeled interactions between several variables indicate that full development, smooth phased development and lumpy development can all be optimal under different market conditions, with each pattern feeding back into inventory levels and lot pricing.

Keywords Real estate development · Sequential investment · Phasing · Inventory

Introduction

Real estate developers that acquire large tracts of land for the purpose of subdividing it into residential lots must make a number of decisions over the course of a project. They may choose to develop the entire site at once or in a series of phases completed

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sequentially over time. In the event phasing is deemed appropriate, the size and timing of each phase must be determined, as well as the number of finished lots to carry in inventory across phases to satisfy uncertain future demand from builders interested in constructing housing in the subdivision. Each of these choices creates risks and opportunities that ultimately affect financial success.

Although phasing and inventory decisions are undoubtedly important to residential land developers, they have rarely been analyzed simultaneously in the extant real estate research. The gap in the literature is notable in light of the recent economic downturn, which has brought newfound attention to the single-family housing production process and the factors that influence development patterns in diverse environments. For example, some have questioned why tens of thousands of lots were completed and held in inventory by developers in markets such as Atlanta, Dallas, Las Vegas, Orlando, and Phoenix leading up to the financial crisis despite the opportunity to mitigate risk through phased development.[1] Examining the factors driving these decisions may provide a better understanding of housing production cycles and help explain the behavior of residential developers forced to balance countervailing economic concerns.

The model presented in this paper addresses these issues by extending the real options framework for real estate investment decision-making to examine the factors that affect both phased development and phased sales (i.e. holding inventory) over time. These decisions are made, and therefore modeled, simultaneously to explain the delivery of lots to the market in the context of large-scale residential development projects. The unique contribution of the model lies in its ability to explore the countervailing effects of economies of scale, pricing power, carrying costs, signaling, demand momentum and demand volatility in settings where they are anticipated to be both prevalent and important.

Modeled interactions between the aforementioned variables indicate that full development, smooth phased development and lumpy development patterns can all be optimal under different market conditions, with each pattern feeding back into inventory levels and lot pricing. Thus, varying markets and projects can display different investment patterns, inventory levels and pricing under rational decision-making. The results help explain why distressed residential subdivisions, full of developed lots that have not yet been converted into housing units, are more common in some markets than others in periods of economic recession. They also offer insight regarding the future of residential land development activity as market conditions improve.

[1] The *Atlanta Journal-Constitution* reported over 150,000 vacant residential lots existed in the Atlanta metropolitan area in 2008 (August 8, 2008), while the *Dallas Business Journal* reported over 100,000 existed in the Dallas-Fort Worth metropolitan area at approximately the same time (August 17, 2008). More recently, inventories of developed lots exceeding 8–10 years of supply have been reported in metropolitan areas throughout Florida (*Jacksonville Business Journal*, November 19, 2010; Orange County Growth Management Department Outlook Report, October 2009). *The Las Vegas Sun* (March 1, 2009) reported developed residential lots selling for less than the replacement cost of the land improvements alone in subdivisions throughout Las Vegas and Phoenix due to dramatic oversupply that existed in 2009.

Before presenting the formal real options model, it is useful to review some of the underlying assumptions upon which it is based. Housing production is viewed as a multistage process where a site must first be acquired and all necessary regulatory entitlements obtained before moving forward with a project. Site improvements are then completed to prepare lots within the subdivision for vertical construction. Housing units can then be erected on the lots and sold to consumers. In many cases, land development and homebuilding activities are completed by different firms because each requires unique skills and involves unique risks. Land development is generally believed to be the riskiest stage of the housing production process because there is no guarantee that regulatory entitlements can be obtained, and even if they can, future demand for residential lots remains uncertain at the time site improvements commence. The model presented in this paper focuses exclusively on phasing and inventory decisions post-entitlement to gain a better understanding of the variables that influence these decisions.[2]

Economy of scale in construction is one of the variables anticipated to affect land development because it is often observed in the real estate industry, at least up to a point. The marginal cost of completing due diligence may decline as the size of a project increases and large subdivisions may benefit from preferential pricing in the procurement of labor, materials and construction management services (Shoup 2008; Gyourko and Rybczynski 2000; Farris 2001). Empirical research suggests economies of scale exist in residential construction as long as sophisticated building technologies are not required to support high density development and managerial capacity is available (Wheaton and Simonton 2007). Thus, one would expect it to be cost effective to develop residential lots in relatively large increments rather than a series of smaller phases unless management capacity becomes strained with increased scale or other countervailing factors exist.

Once lots are completed, carrying costs are expected to provide land developers with an incentive to sell them to prospective homebuilders relatively quickly to reduce the financial burden of inventory. Signaling effects may further accelerate the sales process in situations where construction activity resolves uncertainty about the characteristics of a subdivision and encourages subsequent buyers to pay more for the lots within. This type of behavior has been observed in the residential housing market, indicating that decisions made by developers in one phase of a project affect demand in subsequent phases through signals sent to consumers (Sirmans et al. 1997).[3]

[2] Early theoretical studies completed by Hayashi and Trapani (1978) and Tompkinson (1979) attempt to quantify the benefits of holding inventory in the homebuilding industry when faced with uncertain demand, while considering the offsetting effects of carrying costs. The optimal inventory level for speculative homebuilders is identified in the former study by minimizing lost revenue and in the latter by minimizing construction costs. Demand is assumed exogenous in both models. These studies offer a foundation to begin examining optimal development strategies for residential land developers, but fall short of considering optimal phasing and inventory decisions concurrently in an uncertain economic environment.

[3] An empirical study completed by Sirmans et al. (1997) found evidence that single-family houses sell at a discount in the first phase of a subdivision, with prices increasing over subsequent phases as uncertainty is resolved.

The benefits of building in large phases and holding little inventory, as described above, are anticipated to be tempered when land developers have some degree of pricing power created by the size or strategic location of the parcels being built upon. In such a scenario, developers have the ability to influence lot prices within a submarket by restricting or adding to the number of lots available (Read 1997; Somerville 1999; Wang and Zhou 2006). Developers may therefore choose to complete projects in smaller phases and hold limited inventory to preserve short-run pricing power when carrying costs are high, or alternatively, develop in large phases and hold more inventory when economies of scale are substantial and carrying costs low. Both approaches can potentially be optimal depending upon the interactive effects of the variables discussed thus far.

Market volatility is a final variable that must be taken into account to analyze the complex environment in which phasing and inventory decisions are made. A real option-based valuation model offers an appropriate tool for the task. This type of modeling has advanced rapidly since its initial application to real estate development projects over two decades ago by Titman (1985) and others.[4] Real option theory acknowledges that the owner of a vacant parcel of land has the right, but not the obligation, to develop a property at some point in the future. A real option is valuable in an uncertain economic environment because foregoing development today preserves the opportunity to develop at a later date when additional information is available. Option pricing models built around this basic concept have been used in the of real estate development literature to evaluate the conversion of land on the urban fringe (Capozza and Helsley 1990); time lags between production decisions and the delivery of new space to the market (Majd and Pindyck 1987); flexibility in the timing and intensity of development (Williams 1997; Capozza and Li 1994, 2002); information asymmetries (Grenadier 1999; Childs et al. 2002); and the option to abandon or redevelop a project in the future (Williams 1991; Childs et al. 1996).

A series of related studies utilize real option models to consider the effects of market competition on land values and development timing (Williams 1993; Grenadier 1996; Wang and Zhou 2006).[5] Although the results suggest fear of competitive preemption accelerates real estate development and encourages overbuilding in areas experiencing declining demand, optimal phasing and inventory

[4] Titman's (1985) seminal work examined the option to develop a parcel of land in a two-period setting, where both revenues and construction costs were presumed uncertain and development was anticipated to occur in an all-or-nothing manner. As expected, market volatility was found to increase the value of the development option, thereby increasing the opportunity cost of moving forward with a project immediately.

[5] Williams (1993) emphasized key differences between real options and more traditional financial options by modeling real estate development decisions in an environment where multiple firms facing uncertain demand and limited production capacity simultaneously invest in a land-constrained market knowing their choices will impact the decisions of other developers and ultimately market prices. Grenadier (1996) employed a similar equilibrium framework, allowing for simultaneous or sequential development by two competitive firms, to explain bursts of construction activity in volatile markets and prolonged periods of overbuilding in areas experiencing declining demand. Wang and Zhou (2006) extended the analysis by allowing for both sequential development and competition among multiple firms. The conclusions drawn in the theoretical models described are supported by numerous empirical studies. See Quigg (1993); Holland et al. (2000); Sivitanidou and Sivitanides (2000); Plantiga et al. (2002); Sing and Patel (2001); Cunningham (2006); Schwartz and Torous (2007); Towe et al. (2008), and Bulan et al. (2009).

decisions are not examined concurrently in these models.[6] The model presented in this paper addresses these issues to provide a more complete picture of the residential land development process in situations where real estate developers control large or strategically located parcels of land that afford them some degree of exogenously determined pricing power.

Rather than focusing on competitive interactions among developers, optimal phasing and inventory decisions are concurrently estimated for a developer that controls a single parcel of land capable of accommodating a defined number of residential units. The model acknowledges time lags between production decisions and the completion of developed lots, in addition to flexibility in the timing of development over multiple years and the ability to abandon a project in the future. Economies of scale in construction, carrying costs, signaling effects, demand momentum and market volatility are also incorporated into the analysis to examine how these push and pull factors influence production decisions and undeveloped land values in markets with different characteristics. To the authors' knowledge, the model is the first of its kind to consider all of these variables, while addressing optimal phasing and inventory decisions concurrently.

Economies of scale in construction are found to discourage phasing, which in turn results in higher inventory levels when a developer can preserve pricing power by restricting the supply of developed lots released into the market. Substantial carrying costs may, however, limit the benefits of such an approach and encourage more phasing to take advantage of pricing power without holding excessive amounts of inventory. At the same time, signaling effects initially slow the sale of lots as developers attempt to capture higher prices for higher quantities in later periods after sending signals to the market about the characteristics of a subdivision by gradually releasing lots to the market. Demand volatility simultaneously encourages phasing and inventory to preserve the option to sell lots in the future should market conditions improve. Interactions between all of these variables result in the highest undeveloped land values in volatile markets with low carrying costs, where land developers have the ability to influence prices through strategic phasing and inventory decisions, while benefiting from economies of scale and signaling effects.

The results generated by the model help explain why significant amounts of inventory were held in some markets leading up to the financial crises because economies of scale in construction, low carrying costs and pricing power provided an incentive to build in large phases and release lots gradually over time. Phasing may have proven less attractive in this type of environment because inventory risk was offset by lower marginal production costs and pricing power. While other factors clearly influenced phasing and inventory decisions as well, the interactive

[6] Cortazar and Schwartz (1993) consider phasing and inventory concurrently in an operations management setting by examining the compound real option created by a firm's ability to engage in one stage of a sequential production process in order to create an option to engage in a second stage. They found optimal work-in-progress inventory levels increase with market volatility, despite inventory carrying costs, in order to reduce the risk of production bottlenecks in the event demand for finished goods spikes in the future. While informative, this model is not directly applicable to real estate development because the authors expect inventory to be held only to address inefficiencies in the production process, rather than to take advantage of economies of scale in construction, pricing power, and/or signaling effects. Appropriately specified real estate development models must also accommodate "time-to-build" constraints resulting from lengthy construction cycles, as well as supply constraints that exist when a developer controls a finite amount of land.

effects of the variables included in this model provide useful insight into the decision-making process of residential land developers during this period of time.

Modeling Phasing and Inventory Decisions

Consider a developer who controls a tract of land that can be developed into N finite residential lots. Assume the size of the development and/or the market environment warrants a price effect that is a function of the amount of lots sold within a given time frame. There are up to T phases over which the site can be developed and sold, thus, the developer must decide at the beginning of each phase how much land to develop and how much of the developed land to either sell or hold in inventory in anticipation of future demand.

The market setting described above can be modeled in the standard continuous-time real options framework, where at each time t the developer immediately chooses the percentage of N lots to sell, X_t, out of existing inventory, where the percentage of lots previously developed is held in inventory I_t until it is sold with $X_t \leq I_t$. The developer also chooses the percentage of available land to develop, x_t, subject to the constraints that $x_t \leq J_t$ where $J_t = 1 - \int_{t=o}^{t} x_t$is the amount of land remaining. The developer also has the option to abandon the project at any time. In the event of abandonment at time t, the developer first decides how much available inventory to sell according to the downward sloping demand function and then sells all remaining inventory at a fraction γ of that price and walks away. It is presumed cost prohibitive to hold developed land beyond time T.

The elasticity of demand is assumed to be constant over each phase and the inverse demand function is influenced by land currently sold, as well as the total amount of land sold in all previous phases, and is given by:

$$P(\theta_t, K_t, X_t) = \theta_t X_t^{-\varepsilon}(1 + K_t)^{\beta} \tag{1}$$

where $K_t = \int_{t=o}^{t} X_t$is the percentage of lots sold up to time t and θ_t is a log-normally distributed and mean-reverting random variable. The parameter ε is the inverse of the constant elasticity of demand. The parameter β can represent a signaling effect and provides a mechanism to address information conveyed to prospective buyers during the development process. A positive beta indicates demand for developed land at time t increases with the amount of land sold in all previous phases, a result that would be expected in the presence of signaling effects.[7]

The demand parameter θ is assumed to be log-normally distributed to ensure that it remains positive at all times. It is also assumed to be mean reverting and to have a long-run mean increasing linearly with time to capture the property that demand should revert to an increasing long-run value. In the risk neutral measure, the logarithm of the demand parameter θ is assumed to evolve according to the process:

$$d \ln \theta = \alpha dt + \kappa \Big(\ln \theta + \alpha t - \ln \theta \Big) dt + \sigma dz \tag{2}$$

[7] Note that at any time t, the percentage of the total lots held in inventory plus percentage of the total lots sold plus the percentage of the lots remaining equals 1, i.e., $I_t + J_t + K_t = 1$ ><. Thus, knowing any two of these auxiliary state variables provides the value for the third.

Given the initial value $\ln\theta_0$ the solution at time t is

$$\ln\theta_t = e^{-\kappa t}\ln\theta_0 + \alpha\, t + (1-e^{-\kappa t})\ln\bar{\theta} + \sqrt{\sigma^2(1-e^{-2\kappa t})/(2\kappa)}\, z$$

where z is standard normal and therefore ln θ_t is normally distributed in the risk neutral measure with mean $e^{-\kappa t}\ln\theta_0 + \alpha t + (1-e^{-\kappa t})\ln\bar{\theta}$ and variance $\frac{\sigma^2}{2\kappa}(1-e^{-2\kappa t})$. As t gets large, the mean asymptotically approaches the linear function$\alpha\, t + \ln\bar{\theta}$ with speed determined by the mean reversion parameter κ. Note that greater values of $\bar{\theta}$give larger values for the long run mean; in the following context $\bar{\theta}$is referred to as the long run mean of θ although strictly speaking the mean of θ asymptotically approaches $\bar{\theta}e^{\alpha\, t+\sigma^2/4\kappa}$. With a constant market price of risk λ the drift of θ in the true measure is $\left(\mu_P + \kappa(\ln\bar{\theta} + \alpha\, t - \ln\theta)\right)\theta$ where $\mu_P = \alpha + \sigma\lambda + \sigma^2/2$.[8]

Carrying costs are assumed to be linear and apply to inventory held over time. The cost of holding developed lots in inventory is cNI_t, where c is some positive constant. Total construction costs $C(x_t)$ required to develop the percentage x_t of the land in each phase is a concave function of x, therefore accommodating economies of scale in construction:

$$C(x_t) = (1+\mu_c)^t\left(\alpha_0 + \alpha_1 x_t + \alpha_2 x_t^2\right),\ \text{where } \alpha_0 \geq 0,\ \alpha_1 > 0,\ \alpha_2 < 0 \tag{3}$$

where μ_C is the growth rate in costs. Later a cubic functional form for construction costs is also considered to model the situation in which there are economies of scale only up to a certain point; for very large projects, capacity becomes strained and there may be diseconomies of scale as it becomes more difficult for the developer to hire additional labor and equipment.

In the continuous-time framework, the developer solves a stochastic optimal control problem to find the controls (land to develop and sell: x_t,X_t) that optimize the value of the tract of land. The state at any time is determined by the demand parameter θ_t, the amount of land remaining, J_t, and the amount of land sold, K_t; where the controls steer the state variables according to $dJ = -xdt$ and $dK = Xdt$. Let $V(\theta_t, J_t, K_t; x_t, X_t)$ denote the value given a particular choice x_t,X_t. Standard arguments imply that the optimal value V^* may be found by solving

$$\underset{x,X}{Max}\left[\begin{array}{c}\frac{1}{2}\frac{\partial^2 V}{\partial\theta^2}\theta^2\sigma^2 + \frac{\partial V}{\partial\theta}\theta\left(\alpha + \frac{1}{2}\sigma^2 + \kappa(\ln\bar{\theta} + \alpha t - \ln\theta)\right) - \frac{\partial V}{\partial J}x + \frac{\partial V}{\partial K}X \\ + \frac{\partial V}{\partial t} - C(x) - cNI(J,K) + P(\theta,K,X)NX - rV\end{array}\right] = 0 \tag{4}$$

subject to $x \leq I$ and $X \leq J$.

The boundary condition for V^* at the end of the project is:

$$V^*(\theta, J, K, T) = \underset{X_T \leq I_T}{Max}\left(\theta_T X_T^{-\varepsilon}(1+K_T)^{\beta} N(X_T + \gamma(I_T - X_T))\right);$$

[8] Using Ito's Lemma, the process for θ in the physical measure is $d\theta/\theta = (\mu_P + \kappa(\ln\bar{\theta} + \alpha t - \ln\theta))dt + \sigma\, dz$ ><.

and at any earlier time there may be values (θ^*, J^*, K^*) of the state variables for which it is optimal to abandon. These values must satisfy the smooth pasting and value-matching conditions:

$$\begin{aligned} V^*\left(\theta^*, J^*, K^*, t\right) &= V_A\left(\theta^*, J^*, K^*, t\right) \\ \frac{\partial V^*}{\partial \theta}\left(\theta^*, J^*, K^*, t\right) &= \frac{\partial V_A}{\partial \theta}\left(\theta^*, J^*, K^*, t\right) \\ \frac{\partial V^*}{\partial J}\left(\theta^*, J^*, K^*, t\right) &= \frac{\partial V_A}{\partial \theta}\left(\theta^*, J^*, K^*, t\right) \\ \frac{\partial V^*}{\partial K}\left(\theta^*, J^*, K^*, t\right) &= \frac{\partial V_A}{\partial K}\left(\theta^*, J^*, K^*, t\right) \end{aligned}$$

where V_A is the abandonment value.

This stochastic optimal control problem may be solved numerically using a discrete time setting. To facilitate the numerical computation necessary to solve the model in discrete time, the process for the demand parameter ln θ is approximated by using the standard design for a recombining binomial tree for an Ornstein-Uhlenbeck process. It is assumed that there are up to T phases over which the site can be developed and sold, therefore there are T+1 decision times; at the beginning of each phase the developer chooses x_t, X_t to optimize the present value. At the beginning of the project, the developer selects the optimal amount of land to develop knowing that future choices will be made optimally in all subsequent phases. This dynamic programming problem is solved by first finding the solution over the final phase and sequentially working backward through time.

To illustrate the solution process in discrete time, at time T the developer observes the demand parameter θ_T and selects the optimal amount to sell for each discrete J_T, K_T combination by maximizing the time T value

$$\underset{X_t}{Max}\ V_T(\theta, J_T, K_T) = \theta_T X_T^{-\varepsilon}(1+K_T)^{\beta} N(X_T + \gamma(I_T - X_T)) \tag{5}$$

subject to the constraint that $X_T \leq I_T$. The developer observes the demand parameter $\theta_{T\text{-}1}$ at time T-1 and selects the optimal amounts to develop and sell for each of the auxiliary state variable combinations ($J_{T\text{-}1}$, $K_{T\text{-}1}$), by maximizing

$$\underset{x_{T-1}, X_{T-1}}{Max}\ V_{T-1}(\theta, J_{T-1}, K_{T-1}) = \theta_{T-1} X_{T-1}^{-\varepsilon}(1+K_{T-1})^{\beta} N X_{T-1} - cNI_{T-1} - C(x_{T-1}) + \frac{E_{T-1}\left[V_T^*\right]}{(1+r)^{T-1}} \tag{6}$$

subject to the constraints that $x_{T\text{-}1} \leq J_{T\text{-}1}$, where $J_{T-1} = 1 - (x_1 + \ldots + x_{T-2})$ is the percentage of available land at time T-1 and $X_{T\text{-}1} \leq I_{T\text{-}1}$; V_T^* is the optimal time T value for $J_T = J_{T-1} - x_{T-1}$ and $K_T = K_{T-1} + X_{T-1}$. Continuing in this fashion, the problem is solved by sequentially solving

$$V_t^*(\theta, J_t, K_t) = \max\left(N\theta_t X_t^{-\varepsilon}(1+K_t)^{\beta} - cNI_t - C(x_t) + \frac{E_{t-1}\left[V_t^*\right]}{(1+r)^{t-1}}\right) \tag{7}$$

subject to the constraints $0 \leq X_t \leq I_t$ and $0 \leq x_t \leq J_t$.

The optimal x_0 is found working backward until time 0. Note that $I_t + J_t + K_t = 1$; thus, V_t^* is a function of any two of the three auxiliary state variables. In practice, at each node and for each pair of incoming state variables K_t and J_t, the optimal land to develop and sell (x_t, X_t) are found and stored. These variables are then used to find the optimal (x_{t-1}, X_{t-1}) at a node one period earlier with incoming state variables K_{t-1} and J_{t-1}. After this dynamic programming problem is solved and the optimal amount x_0 to develop over the first period is found, the time zero expectation of optimal inventory, development, sales and price for each time period is identified by working forward through the tree, using real rather than risk-neutral probabilities.[9]

Results and Sensitivity Analysis

The numerical procedures described above and the base case parameters presented in Table 1 are used in this section to estimate optimal phasing, sales and inventory decisions for a hypothetical parcel of land capable of supporting N=100 residential lots.[10] It is assumed lots can be sold in one unit increments, but must be developed in units of five. The demand function is presumed to be downward sloping with the elasticity of demand set at 6. Therefore, the elasticity of the inverse demand curve in Eq. 1 is 0.167.[11] For the stochastic demand curve, $\theta_0 = \theta = 1$, i.e., the initial value is set to the initial equilibrium value, and the volatility of the stochastic demand parameter is set at 0.15. The growth rate in the price over time (affected through θ) is set at μ_P=0.03, and κ=0.15 is used to control the speed of the mean-reversion process. Risk-neutral growth rates in process are modeled as a function of the market price of risk and is set at λ=0.333, which corresponds to a 5% risk premium for the risk in stochastic demand (prices). The elasticity of the signaling effect β is initially set at zero. With these parameters, the price of each lot at time zero is $1 if all 100 lots are sold at once, regardless of the elasticity parameter. Market power is reflected in higher prices if less than 100 lots are sold in a period. The degree of such power is determined by the price elasticity parameter.

Construction costs are modeled as an increasing quadratic function of the number of lots produced in each phase. In the base case, the fixed cost is set at zero and the linear variable cost is set at $100 to reflect $1 in construction costs per lot when N= 100. The quadratic constant term is −$20, representing $0.20 in cost savings per lot from economies of scale when 100% of the lots are developed at once. These values, and therefore construction costs, are assumed to grow 3% each period. Inventory

[9] The incoming inventories at time t=1 nodes are I_1=x_0 and J_1=1-x_0; for each of these nodes, the stored optimal values of x_1 and X_1 are returned and the incoming inventories at time t=2 are updated $I_2 = I_1 - X_1 + x_1$ >< and $J_2 = J_1 - x_1$ ><. Given these inventories, for each node at time t=2 the stored optimal values of x_2 and X_2 are returned and the inventories at time t=3 are updated, etc. Because of the path dependency of the optimal strategies, the number of states grows exponentially with time: if the tree is constructed with k time steps over each phase then there will be $(k+1)^t$ >< states at time t. To compute the expected amounts to sell and develop each phase the true probabilities of the states and in particular the true probabilities of the nodes in the tree for ln θ must be found.

[10] Units represent a percentage of a project of any size.

[11] Elasticities for *new* residential construction are historically between 4 and 13 according to existing research. See Malpezzi and Maclennan (2001) and Green et al. (2005).

Table 1 Base case parameters

Parameters and descriptions	Base case value
Parameters for the stochastic demand function	
θ_0 is the stochastic normal demand parameter at time zero	θ_0=1.00
θ is the initial long-run mean of normal demand parameter at time zero	$\theta = 1$
ε is the inverse of the demand elasticity	ε=.167
β is the price elasticity of the signaling effect	β=0
μ_P is the growth rate of demand (price)	μ=.03
κ is the reversion rates of demand	κ=.15
σ is the demand volatility rate	σ=.15
λ is the market price of risk	λ=.333
Construction and holding cost parameters	
C is construction cost function	$C(x_0) = 100x_0 - 20x_0^2$
μ_C is the growth rate of construction costs	μ_C=.03
c is a fixed positive constant representing inventory carrying costs	c=.10
Other parameters	
N is the total number of lots that can be developed	N=100
Δt is the length of time to complete each phase in years	Δt=1
T total periods in years	T=7
γ is the fraction of the price received on abandoned developed units	γ=0
r is the risk-free rate	r=5%

carrying costs are presumed to be linear and are set at \$0.10 per lot developed, but not yet sold. The time required to complete each phase is anticipated to be 1 year irrespective of the number of lots developed. The parameter γ is set at 0, indicating that the developer loses any unsold inventory upon abandonment or any inventory remaining at the end of time horizon T=7 years. The risk free-rate of return is 5%.

The results for the base case parameters are presented in the center column and row of panel B in Table 2. For the base case, the value of the land is \$28.08 or approximately 28% of the lot revenues if they were are all sold at time zero. With phased sales, the overall weighted average price is \$1.42 per lot. When the base-case results are varied across inventory costs (the results over the center column), average lot price runs from 1.42 to 1.51. Phasing is not warranted when inventory costs are low enough to allow the developer to take advantage of economies of scale. When inventory costs are high, c=.20, phasing begins to take place, though a large initial investment is still made. For this case, the option to sell some lots at potentially high prices in the future causes some development of lots after year 2 and inventories are kept near zero as most of the lots are sold in year 1. When inventory costs are lower, lots are sold in phases over time to preserve pricing power, but over 80% of the sales are still expected to occur in the first 2–3 years.

Moving across the columns of Table 2, demand elasticity increases (the inverse of demand elasticity decreases), and therefore pricing power (competition) decreases (increases). As pricing power decreases, it is not surprising that the value of the land decreases (a 69% decrease as elasticity goes from 3 to 9 in the middle column) as does the average selling price of the lots (a 30% decrease as elasticity goes from 3 to 9 in the middle column). Generally, increased pricing power provides an incentive to phase the sale of lots over a longer duration in order to sell in smaller quantities at higher prices. This in turn creates an incentive to phase development over time to reduce inventory and its associated holding cost (phasing of development occurs in the higher elasticity, higher holding cost cases). Most of the sales are made in the first 4 years when elasticity is set at 3 (a less competitive market), whereas most are made in the first 2–3 years when elasticity is set at 9 (a more competitive market).

The results show that the optimal solution, including the expected level of inventory, is highly affected by the cost of holding inventory and the pricing power in the market. It is interesting to note that higher inventory carrying costs don't always speed sales and reduce the average price of lots, because as inventory costs increase, lots are then developed and sold in phases; thus, additional profits from pricing can be captured. This shows that areas with high holding costs (e.g., property taxes) will pass on these costs to buyers through phased sales at higher prices. More competitive markets therefore tend to have less phasing in development and lower sales prices.

These results may help explain why many residential land developers held significant amounts of inventory in markets throughout the southeastern and southwestern United States in the years leading up to the financial crises. Economies of scale in construction, coupled with low carrying costs and the ability to acquire large tracts of developable land, encouraged developers to build in large phases and release the inventory to the market relatively slowly over time to restrict supply in some submarkets. Phased development proved less attractive in these rapidly growing areas because the risk of holding inventory was perceived to be modest in comparison to the benefits derived from lower marginal production costs and pricing power. Although other factors undoubtedly influenced production decisions as well, the interactive effects of these variables provide useful insight into the decision-making process of profit maximizing land developers responding to market forces in the mid 2000s.

Table 3 presents the results of the model after varying inventory costs and economies of scale in construction to further illustrates the conditions that influence how well the short-run supply of residential lots tracks demand via phasing and inventory decisions. Outputs generated by the base case parameters are again presented in the center column/row for comparative purposes. As one moves right across the table, increasing economies of scale in construction produce higher land values, lower average lot sales prices, less phasing, and higher inventory levels. Greater inventories encourage faster lot sales to reduce the cost of holding inventory. Faster sales then lead to lower average lot sales prices, but the economies of scale in development more than offset these effects to produce higher land values. Again, phased development only occurs when there is a combination of adequate pricing power (higher elasticity) and high inventory costs. In these cases, it is better to postpone some development to wait for a high price environment and/or reduce the high cost of inventory, each of which overcomes the cost advantage of building all of

Table 2 Results for the base case varied over price elasticity and inventory costs

	ε=1/3				ε=1/6				ε=1/9			
Panel A: c=.05												
	V=75.12				V=33.25				V=23.92			
	Overall average price = 2.27				Overall average price = 1.51				Overall average price = 1.30			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	1.0000	0	0	0	1.0000	0	0	0	1.0000	0	0	0
1	0	0.2799	0.7201	1.6425	0	0.4362	0.5638	1.2348	0	0.5668	0.4332	1.1458
2	0	0.2200	0.5001	1.8987	0	0.2580	0.3058	1.4385	0	0.2544	0.1788	1.3376
3	0	0.1678	0.3324	2.2049	0	0.1481	0.1576	1.6758	0	0.1077	0.0711	1.5640
4	0	0.1270	0.2054	2.5547	0	0.0802	0.0774	1.9661	0	0.0437	0.0273	1.8339
5	0	0.0923	0.1131	2.9922	0	0.0429	0.0345	2.3027	0	0.0166	0.0107	2.1589
6	0	0.0664	0.0467	3.5093	0	0.0222	0.0123	2.7177	0	0.0084	0.0023	2.2082
7	0	0.0467	0.0000	4.1515	0	0.0123	0.0000	3.0660	0	0.0023	0.0000	1.9812
Panel B: c=.10												
	V=66.68				V=28.08				V=20.86			
	Overall average price = 1.75				Overall average price = 1.42				Overall average price = 1.23			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.8000	0	0	0	1.0000	0	0	0	1.0000	0	0	0
1	0	0.3282	0.4718	1.5587	0	0.5641	0.4359	1.1836	0	0.7173	0.2827	1.1167
2	0	0.2229	0.2490	1.8919	0	0.2431	0.1928	1.4541	0	0.1991	0.0836	1.3747
3	0.0066	0.1510	0.0980	2.2852	0	0.1057	0.0871	1.7724	0	0.0575	0.0261	1.6760
4	0.0936	0.1022	0.0024	2.7538	0	0.0477	0.0394	2.1386	0	0.0173	0.0088	2.0350
5	0.0530	0.0867	0.0094	3.0536	0	0.0216	0.0178	2.5697	0	0.0087	0.0001	2.1520

6	0.0466	0.0598	0.0026	3.6224	0	0.0102	0.0076	3.0226	0	0.0001	0.0000	1.5703
7	0	0.0492	0.0000	4.0387	0	0.0076	0.0000	2.9331	0	0	0.0000	N\A
Panel C: c=.20												
	V=65.59				V=24.00				V=18.33			
	Overall average price = 2.36				Overall average price = 1.47				Overall average price = 1.15			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.2000	0	0	0	0.7500	0	0	0	1.0000	0	0	0
1	0.2033	0.1999	0.0001	1.8410	0	0.6009	0.1491	1.1719	0	0.8752	0.1248	1.0927
2	0.1824	0.2033	0.0001	1.9534	0.0878	0.1488	0.0003	1.5782	0	0.1049	0.0199	1.4755
3	0.1459	0.1824	0.0001	2.1475	0.0611	0.0882	0.0000	1.8305	0	0.0199	0.0000	1.8827
4	0.1092	0.1460	0.0001	2.4364	0.0497	0.0609	0.0001	2.0602	0	0	0.0000	N/A
5	0.0966	0.1091	0.0002	2.8372	0.0446	0.0482	0.0017	2.2371	0	0	0.0000	N/A
6	0.0610	0.0968	0.0000	3.1004	0.0062	0.0380	0.0082	2.4777	0	0	0.0000	N/A
7	0	0.0610	0.0000	3.8232	0	0.0144	0.0000	3.0078	0	0	0.0000	N/A

The developer chooses X_t, the percentage of lots sold out of existing inventory. I_t is the percentage of lots held in inventory. The developer also chooses, x_t, the percentage of available land developed. Average values are expected values over the theta nodes at each time

Table 3 Results varied over construction cost economies of scale and inventory costs

	$C(x_0) = 100x_0 - 10x_0^2$				$C(x_0) = 100x_0 - 20x_0^2$				$C(x_0) = 100x_0 - 30x_0^2$			
Panel A: c=.05												
	V=23.50				V=33.25				V=43.25			
	Overall average price = 1.59				Overall average price = 1.51				Overall average price = 1.51			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.8500	0	0	0	1.0000	0	0	0	1.0000	0	0	0
1	0	0.4069	0.4431	1.2491	0	0.4362	0.5638	1.2348	0	0.4362	0.5638	1.2348
2	0.0010	0.2482	0.1949	1.4474	0	0.2580	0.3058	1.4385	0	0.2580	0.3058	1.4385
3	0.0476	0.1478	0.0481	1.6769	0	0.1481	0.1576	1.6758	0	0.1481	0.1576	1.6758
4	0.0550	0.0853	0.0105	1.9419	0	0.0802	0.0774	1.9661	0	0.0802	0.0774	1.9661
5	0.0392	0.0549	0.0105	2.2129	0	0.0429	0.0345	2.3027	0	0.0429	0.0345	2.3027
6	0.0049	0.0344	0.0153	2.5344	0	0.0222	0.0123	2.7177	0	0.0222	0.0123	2.7177
7	0	0.0202	0.0000	2.9038	0	0.0123	0.0000	3.0660	0	0.0123	0.0000	3.0660
Panel B: c=.10												
	V=22.50				V=28.08				V=38.08			
	Overall average price = 1.66				Overall average price = 1.42				Overall average price = 1.42			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.2500	0	0	0	1.0000	0	0	0	1.0000	0	0	0
1	0.2477	0.2411	0.0089	1.3643	0	0.5641	0.4359	1.1836	0	0.5641	0.4359	1.1836
2	0.1747	0.2500	0.0066	1.4558	0	0.2431	0.1928	1.4541	0	0.2431	0.1928	1.4541
3	0.1262	0.1789	0.0024	1.6262	0	0.1057	0.0871	1.7724	0	0.1057	0.0871	1.7724
4	0.0907	0.1269	0.0017	1.8227	0	0.0477	0.0394	2.1386	0	0.0477	0.0394	2.1386
5	0.0653	0.0901	0.0023	2.0509	0	0.0216	0.0178	2.5697	0	0.0216	0.0178	2.5697

6	0.0281	0.0589	0.0087	2.3779	0	0.0102	0.0076	3.0226	0	0.0102	0.0076	3.0226
7	0	0.0367	−0.0000	2.7584	0	0.0076	0.0000	2.9331	0	0.0076	0.0000	2.9331
Panel C: c=.20												
	V=22.08				V=24.00				V=32.56			
	Overall average price = 1.67				Overall average price = 1.47				Overall average price = 1.29			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.2500	0	0	0	0.7500	0	0	0	1.0000	0	0	0
1	0.2202	0.2495	0.0005	1.3566	0	0.6009	0.1491	1.1719	0	0.7453	0.2547	1.1308
2	0.1805	0.2203	0.0003	1.4889	0.0878	0.1488	0.0003	1.5782	0	0.1749	0.0798	1.5359
3	0.1375	0.1806	0.0002	1.6287	0.0611	0.0882	0.0000	1.8305	0	0.0499	0.0300	2.0074
4	0.0923	0.1376	0.0001	1.7965	0.0497	0.0609	0.0001	2.0602	0	0.0200	0.0100	2.4649
5	0.0641	0.0923	0.0001	2.0381	0.0446	0.0482	0.0017	2.2371	0	0.0100	0.0000	2.8997
6	0.0399	0.0605	0.0036	2.2963	0.0062	0.0380	0.0082	2.4777	0	0	0.0000	N/A
7	0	0.0435	0.0000	2.6767	0	0.0144	0.0000	3.0078	0	0	0.0000	N/A

The developer chooses X_t, the percentage of lots sold out of existing inventory. I_t is the percentage of lots held in inventory. The developer also chooses, x_t, the percentage of available land developed. Average values are expected values over the theta nodes at each time

the lots upfront. The results suggest markets with significant amounts of inventory do not exhibit the requisite combination of these variables to place economic pressure of developers to phase production.

Table 4 shows results where construction costs are linear in development, thus no economies of scale exist. In this case, phasing of development is always optimal for moderate levels of inventory costs. Building and sales are essentially synchronized so that inventories are kept near zero. In the homebuilding industry, as opposed to lot development, synchronized development and sales are the norm due to the lack of economies of scales. On the other hand, lot development often exhibits development phasing due to economies of scale in construction.

Construction costs are next modeled with economies of scale to a certain point, beyond which additional development actually increases average cost. This would be the case in projects or markets where a large level of development increases the costs to hire additional labor and equipment. In this spirit, a third term is added to the cost function, $C(x_0) = 100x_0 - 60x_0^2 + 60x_0^3$, to provide for diseconomies of scale beyond development of over 50% of the project in any one period. The results, shown in Table 5, indicate that phasing development is optimal to avoid diseconomies of scale and development can be quite lumpy across years. This lumpiness occurs because it is optimal to develop a certain number of lots to take advantage of the initial economies of scale and then wait as these lots are sold and inventory is depleted across periods before additional development is warranted. Development can occur between these "high" development periods, but it only takes place in very high price/demand environments.

The parameter β in Eq. 1 is modeled to represent a signaling effect and provides a mechanism to address information conveyed to prospective buyers during the development process. A positive beta indicates demand for developed land at time t increases with the amount of land sold in all previous phases, as additional lot sales signal to potential customers that the development will be successful. Table 6 presents results when β=.33.[12] These results can be directly compared to the results in Table 2, which are identical in parameters and variations thereon with the exception that β=0 in the previous table. The signaling effect produces higher average sales prices as the development is completed resulting in higher land values. A positive signaling effect generally promotes somewhat slower sales in the early years in order to establish positive signals to take advantage of higher prices as the development is sold out. This indicates a strategy of low prices to early buyers to entice sales with the idea of taking advantage of the additional pricing power in later years as uncertainty is reduced and sales prices increase. Additionally, more phased investment (i.e. slower development in cases where there are high inventory costs) is used to offset the somewhat slower sales in order to hold down inventory costs.

Although Table 6 indicates signaling has some effect on optimal phasing and inventory decisions, it is not as profound as one might think due to countervailing forces. Selling developed lots for conversion into housing units provides the market with information about the characteristics of a subdivision that increases the value of the remaining lots held by the land developer. However, each lot released from inventory is no longer available for sale in subsequent periods at the resulting higher

[12] Other levels of β were analyzed with similar results.

Table 4 Results with no economies of scale varied over price elasticity

$C(x_0) = 100x_0$												
c=.10												
	ε=1/3				ε=1/6				ε=1/9			
t	V=62.9059				V=21.0784				V=11.6087			
	Overall average price = 2.28				Overall average price = 1.66				Overall average price = 1.50			
0	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
1	0.2000	0	0	0	0.2000	0	0	0	0.2000	0	0	0
2	0.1943	0.1995	0.0005	1.8420	0.2071	0.1957	0.0043	1.4122	0.2198	0.1940	0.0059	1.2913
3	0.1699	0.1935	0.0013	1.9855	0.1862	0.2089	0.0025	1.5001	0.1898	0.2222	0.0036	1.3696
4	0.1475	0.1707	0.0006	2.1928	0.1491	0.1866	0.0021	1.6162	0.1404	0.1905	0.0028	1.4741
5	0.1219	0.1475	0.0006	2.4268	0.1063	0.1494	0.0018	1.7674	0.0979	0.1417	0.0015	1.6115
6	0.0987	0.1200	0.0025	2.7440	0.0755	0.1066	0.0014	1.9783	0.0638	0.0969	0.0027	1.7819
7	0.0648	0.0947	0.0064	3.1058	0.0496	0.0725	0.0044	2.2351	0.0380	0.0613	0.0052	1.9979

The developer chooses X_t, the percentage of lots sold out of existing inventory. I_t is the percentage of lots held in inventory. The developer also chooses, x_t, the percentage of available land developed. Average values are expected values over the theta nodes at each time

Table 5 Results construction costs when there are economies initially and then diseconomies of scale varied over price elasticity

$C(x_0) = 100x_0 - 60{x_0}^2 + 60{x_0}^3$												
c=.10												
	ε=1/3				ε=1/6				ε=1/9			
	V=71.54				V=29.68				V=20.44			
	Overall average price = 2.32				Overall average price = 1.56				Overall average price = 1.34			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.4000	0	0	0	0.4000	0	0	0	0.4500	0	0	0
1	0.0086	0.2430	0.1570	1.7225	0.3537	0.3860	0.0140	1.2613	0.3989	0.4371	0.0129	1.1798
2	0.3301	0.1648	0.0007	2.0957	0.0383	0.2639	0.1039	1.4353	0.0338	0.3287	0.0830	1.3025
3	0.0026	0.2007	0.1301	2.0764	0.1601	0.1302	0.0120	1.7215	0.0821	0.1086	0.0082	1.5722
4	0.2507	0.1312	0.0015	2.5293	0.0215	0.1144	0.0577	1.8560	0.0191	0.0702	0.0201	1.7577
5	0.0040	0.1234	0.1288	2.7103	0.0212	0.0572	0.0220	2.2128	0.0089	0.0307	0.0086	2.0662
6	0.0036	0.0828	0.0500	3.2523	0.0027	0.0301	0.0131	2.6447	0.0025	0.0151	0.0025	2.1528
7	0	0.0535	0.0000	3.9497	0	0.0158	0.0000	3.0456	0	0.0050	0.0000	1.9739

The developer chooses X_t, the percentage of lots sold out of existing inventory. I_t is the percentage of lots held in inventory. The developer also chooses, x_t, the percentage of available land developed Average values are expected values over the theta nodes at each time price. The developer must therefore sell enough lots to trigger signaling effects, while retaining as many lots as possible to benefit from heightened consumer demand in the future. This result is a “wait-and-see” approach that slows rather than accelerates sales.

Table 7 varies volatility across price elasticity levels at the higher inventory cost level of c=.20.[13] Generally the results show that for higher levels of volatility, average sales prices are higher, resulting in higher land values. Additional volatility also increases the value of the option to wait, creating an incentive to both phase development over time and to sell off the project more slowly to preserve the ability to take advantage of the ability to sell at high prices. When phasing occurs, sales and development are generally synchronized to hold down expected inventories. The result is consistent with existing real options research and clearly illustrates the impact of uncertainty on production decisions.

The initial equilibrium value of demand parameter θ in Eq. 1 is next varied to represent a market that is initially above $\theta_0 \theta$or below $\theta_0 \theta$ the long run equilibrium, where θ_0=1.00. In this way a market that has expected positive or negative price and demand momentum can be modeled. Tables 8 shows the results when the initial value is 50% below the equilibrium, $\theta = 1.50$ (expected positive price momentum) and Table 9, shows the results when the initial value is 50% above the equilibrium, $\theta = .50$ (expected negative price momentum). The results are again varied across price elasticities and inventory costs.

Relative to the base case results in Table 2, when there is positive price momentum, Table 8 shows that there is more phasing and slower sales over time by the developer in order to take advantage of the rising prices. This effect holds in all elasticity and inventory cost scenarios. Average prices for lots are generally 30–40% higher than the base case due to increasing prices over time and the market power captured by phasing sales over time. Land values are also 30–40% higher. This economic environment is reflective of many markets in the go-go days of the mid-2000s where large tracts of land were purchased and developed to capture rising prices. Yet, many land developers built out subdivisions rather rapidly and held inventory during this time period rather than phasing production as predicted by the model. The outcome is potentially attributable to extreme economies of scale in construction made possible by advantageous financing for large land developers engaging in large projects at that point in time. Ready access to capital at extraordinarily competitive rates most likely encouraged developers to move forward with projects despite the potential benefits of phasing.

Relative to the base case results in Table 2, when there is negative price momentum Table 9 shows that there is also more phasing, but often with less than 100% of the lots developed. Most of the lots are developed and sold in the early years to take advantage of prices that are expected to fall. This effect holds in all elasticity and inventory cost scenarios. Average prices for lots are not significantly different from the base case as developers build fewer lots and phase sales over time to preserve prices in the face of negative momentum. Land values however are 50–70% lower. This is reflective of many markets *after* the home price bubble days of the mid-2000s.

[13] This inventory cost is chosen because it best demonstrates the effects of varying the volatility level.

Table 6 Results for case with β=.33 representing the signaling effect varied over price elasticity and inventory costs

	ε=1/3				ε=1/6				ε=1/9			
Panel A: c=.05												
	V=95.2864				V=46.5294				V=34.7011			
	Overall average price = 2.5895				Overall average price = 1.7538				Overall average price = 1.5140			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	1.0000	0	0	0	1.0000	0	0	0	1.0000	0	0	0
1	0	0.2814	0.7186	1.6400	0	0.3637	0.6363	1.2729	0	0.4255	0.5745	1.1829
2	0	0.2178	0.5007	2.0698	0	0.2592	0.3770	1.5944	0	0.2852	0.2893	1.4865
3	0	0.1669	0.3339	2.5283	0	0.1687	0.2084	1.9261	0	0.1588	0.1305	1.7899
4	0	0.1264	0.2075	3.0335	0	0.1026	0.1058	2.2861	0	0.0767	0.0538	2.1152
5	0	0.0932	0.1142	3.6250	0	0.0584	0.0474	2.7009	0	0.0338	0.0200	2.4850
6	0	0.0668	0.0474	4.3304	0	0.0312	0.0161	3.1937	0	0.0138	0.0062	2.8482
7	0	0.0474	0.0000	5.1544	0	0.0161	0.0000	3.7527	0	0.0062	0.0000	2.7315
Panel B: c=.10												
t	V=86.5304				V=39.9846				V=29.7264			
	Overall average price = 2.5306				Overall average price = 1.6618				Overall average price = 1.4307			
0	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
1	0.9000	0	0	0	1.0000	0	0	0	1.0000	0	0	0
2	0	0.3308	0.5692	1.5549	0	0.4541	0.5459	1.2271	0	0.5273	0.4727	1.1555
3	0	0.2275	0.3417	2.0676	0	0.2668	0.2791	1.6219	0	0.2874	0.1853	1.5197
4	0	0.1576	0.1841	2.6139	0	0.1451	0.1341	2.0146	0	0.1194	0.0659	1.8829
5	0.0031	0.1086	0.0756	3.2244	0	0.0725	0.0616	2.4551	0	0.0434	0.0225	2.2754
6	0.0546	0.0749	0.0037	3.9213	0	0.0352	0.0264	2.9563	0	0.0148	0.0077	2.7305
7	0.0424	0.0530	0.0054	4.6574	0	0.0166	0.0099	3.5689	0	0.0072	0.0005	2.7024

Panel C: c=.20												
	V=85.0139				V=35.9428				V=24.8105			
	Overall average price = 2.6398				Overall average price = 1.8019				Overall average price =1.4434			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.2500	0	0	0	0.3500	0	0	0	0.8000	0	0	0
1	0.2082	0.2500	0	1.7088	0.2319	0.3493	0.0007	1.2827	0	0.6000	0.2000	1.1394
2	0.1540	0.2082	0.000	2.0871	0.1712	0.2325	0.0000	1.6196	0.0878	0.1996	0.0004	1.6074
3	0.1457	0.1537	0.0002	2.5773	0.1078	0.1712	0.0001	1.9079	0.0520	0.0882	0.0000	1.9431
4	0.1010	0.1458	0.0001	2.8625	0.0655	0.1079	0.0000	2.2508	0.0487	0.0517	0.0003	2.2093
5	0.0890	0.0999	0.0012	3.5033	0.0513	0.0655	0.0000	2.6453	0.0109	0.0414	0.0076	2.4027
6	0.0522	0.0858	0.0044	3.9868	0.0220	0.0456	0.0057	2.9456	0.0005	0.0164	0.0021	2.8522
7	0	0.0566	0.0000	4.8146	0	0.0277	0.0000	3.5974	0	0.0026	0.0000	2.6532

The developer chooses X_t, the percentage of lots sold out of existing inventory. I_t is the percentage of lots held in inventory. The developer also chooses, x_t, the percentage of available land developed. Average values are expected values over the theta nodes at each time

Table 7 Results varied over volatility

t	ε=1/3				ε=1/6				ε=1/9			
Panel A: σ=.075 c=.20												
	V=62.24				V=22.37				V=17.50			
	Overall average price = 1.96				Overall average price = 1.28				Overall average price = 1.07			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.2500	0	0	0	0.8000	0	0	0	1.0000	0	0	0
1	0.1996	0.2500	0	1.6181	0	0.6510	0.1490	1.0949	0	0.8851	0.1149	1.0333
2	0.1509	0.1996	0	1.7712	0.0571	0.1490	0.0000	1.4215	0	0.0990	0.0159	1.3383
3	0.1478	0.1509	0.0000	1.9681	0.0500	0.0571	0.0000	1.6963	0	0.0159	0.0000	1.6693
4	0.1000	0.1478	0.0000	2.0020	0.0500	0.0500	0.0000	1.7430	0	0	0.0003	N/A
5	0.0993	0.1000	0.0000	2.2957	0.0429	0.0472	0.0028	1.7795	0	0	0.0076	N/A
6	0.0523	0.0993	0.0000	2.3185	0.0000	0.0357	0.0099	1.9051	0	0	0.0021	N/A
7	0	0.0523	0.0000	2.8945	0	0.0100	0.0000	2.3153	0	0	0.0000	N/A
Panel B: σ=.15 c=.20												
	V=65.59				V=24.00				V=18.33			
	Overall average price = 2.36				Overall average price = 1.47				Overall average price = 1.15			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.2000	0	0	0	0.7500	0	0	0	1.0000	0	0	0
1	0.2033	0.1999	0.0001	1.8410	0	0.6009	0.1491	1.1719	0	0.8752	0.1248	1.0927
2	0.1824	0.2033	0.0001	1.9534	0.0878	0.1488	0.0003	1.5782	0	0.1049	0.0199	1.4755
3	0.1459	0.1824	0.0001	2.1475	0.0611	0.0882	0.0000	1.8305	0	0.0199	0.0000	1.8827
4	0.1092	0.1460	0.0001	2.4364	0.0497	0.0609	0.0001	2.0602	0	0	0.0000	N/A
5	0.0966	0.1091	0.0002	2.8372	0.0446	0.0482	0.0017	2.2371	0	0	0.0000	N/A

6	0.0610	0.0968	0.0000	3.1004	0.0062	0.0380	0.0082	2.4777	0	0	0.0000	N/A
7	0	0.0610	0.0000	3.8232	0	0.0144	0.0000	3.0078	0	0	0.0000	N/A
Panel C: σ=.225 c=.20												
	V=71.60				V=28.86				V=20.62			
	Overall average price = 2.90				Overall average price = 2.01				Overall average price = 1.75			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.2000	0	0	0	0.2000	0	0	0	0.5000	0	0	0
1	0.1822	0.1985	0.0015	1.9558	0.2880	0.1987	0.0013	1.4940	0.5730	0.4990	0.0010	1.5952
2	0.1747	0.1833	0.0005	2.2602	0.1759	0.2652	0.0241	1.6292	0.1997	0.4963	0.0777	1.5037
3	0.1541	0.1748	0.0004	2.5498	0.1322	0.1989	0.0011	1.8935	0.0887	0.2384	0.0390	1.8056
4	0.1254	0.1541	0.0005	2.9349	0.0958	0.1326	0.0007	2.2463	0.0447	0.1114	0.0162	2.3034
5	0.0938	0.1238	0.0021	3.4959	0.0644	0.0945	0.0020	2.6496	0.0180	0.0549	0.0060	2.3333
6	0.0663	0.0943	0.0015	4.2598	0.0319	0.0607	0.0058	3.1918	0.0103	0.0216	0.0025	2.2178
7	0	0.0678	0.0000	5.2386	0	0.0376	0.0000	3.7525	0	0.0128	0.0000	2.5502

The developer chooses X_t, the percentage of lots sold out of existing inventory. I_t is the percentage of lots held in inventory. The developer also chooses, x_t, the percentage of available land developed. Average values are expected values over the theta nodes at each time

Table 8 Results for Case With Positive Momentum in Demand $\theta = 1.50$ Varied Over Price Elasticity and Inventory Costs

	ε=1/3				ε=1/6				ε=1/9			
Panel A: c=.05												
	V=97.98				V=45.03				V=31.66			
	Overall average price = 2.85				Overall average price = 2.00				Overall average price = 1.74			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.3500	0	0	0	0.3500	0	0	0	0.3000	0	0	0
1	0	0.1826	0.1674	2.0050	0.0602	0.2033	0.1467	1.4866	0.2346	0.2347	0.0653	1.3426
2	0.3004	0.1628	0.0046	2.3387	0.2876	0.1739	0.0330	1.7115	0.1731	0.2194	0.0804	1.5148
3	0.0064	0.1703	0.1346	2.5367	0.0933	0.2053	0.1153	1.8405	0.1937	0.1981	0.0555	1.7012
4	0.3389	0.1385	0.0025	2.9825	0.1880	0.1549	0.0537	2.1079	0.0827	0.1579	0.0913	1.9059
5	0.0041	0.1465	0.1949	3.1580	0.0189	0.1318	0.1099	2.3585	0.0122	0.1056	0.0684	2.1931
6	0.0001	0.1141	0.0849	3.7048	0.0014	0.0831	0.0457	2.7687	0.0023	0.0559	0.0246	2.5740
7	0	0.0851	0.0000	4.3909	0	0.0471	0.0000	3.2845	0	0.0269	0.0000	2.9350
Panel B: c=.10												
	V=95.90				V=43.17				V=30.11			
	Overall average price = 2.87				Overall average price = 1.98				Overall average price = 1.72			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.1500	0	0	0	0.2000	0	0	0	0.2000	0	0	0
1	0.1789	0.1498	0.0002	2.1443	0.2098	0.1920	0.0080	1.4997	0.2684	0.1921	0.0079	1.3681
2	0.2012	0.1783	0.0008	2.2669	0.2640	0.2116	0.0062	1.6588	0.2770	0.2674	0.0089	1.4859
3	0.1814	0.1753	0.0267	2.5158	0.1051	0.2051	0.0651	1.8376	0.1113	0.2357	0.0502	1.6595
4	0.1331	0.1578	0.0502	2.8522	0.1201	0.1472	0.0230	2.1380	0.0760	0.1390	0.0226	1.9474
5	0.1062	0.1365	0.0469	3.2486	0.0637	0.1182	0.0249	2.4010	0.0532	0.0871	0.0115	2.2267

6	0.0491	0.1138	0.0394	3.7264	0.0366	0.0758	0.0129	2.8074	0.0126	0.0530	0.0117	2.5578
7	0	0.0885	0.0000	4.3637	0	0.0495	0.0000	3.2519	0	0.0242	0.0000	3.0102
Panel C: c=.20												
	V=95.7505				V=42.81				V=29.59			
	Overall average price = 2.87				Overall average price = 1.99				Overall average price = 1.73			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.1500	0	0	0	0.2000	0	0	0	0.2000	0	0	0
1	0.1789	0.1500	0	2.1438	0.1822	0.1995	0.0005	1.4900	0.2522	0.1997	0.0003	1.3623
2	0.1671	0.1788	0.0001	2.2654	0.2009	0.1827	0.0000	1.6974	0.2165	0.2522	0.0004	1.4990
3	0.1539	0.1672	0.0000	2.5633	0.1618	0.2009	0.0000	1.8450	0.1556	0.2167	0.0001	1.6762
4	0.1454	0.1536	0.0001	2.8762	0.1220	0.1618	0.0001	2.0915	0.0895	0.1557	0.0001	1.9058
5	0.1098	0.1457	0.0003	3.1780	0.0776	0.1220	0.0000	2.3849	0.0610	0.0894	0.0002	2.2240
6	0.0948	0.1089	0.0010	3.7818	0.0549	0.0767	0.0010	2.8015	0.0239	0.0553	0.0060	2.5174
7	0	0.0958	0.0000	4.2239	0	0.0558	0.0000	3.1514	0	0.0298	0.0000	2.9657

The developer chooses X_t, the percentage of lots sold out of existing inventory. I_t is the percentage of lots held in inventory. The developer also chooses, x_t, the percentage of available land developed. Average values are expected values over the theta nodes at each time

Table 9 Results for case with negative momentum in demand $\theta = .50$ varied over price elasticity and inventory costs

	ε=1/3				ε=1/6				ε=1/9			
Panel A: c=.05												
	V=37.56				V=10.47				V=5.03			
	Overall average price=1.78				Overall average price=1.30				Overall average price=1.17			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.4000	0	0	0	0.4000	0	0	0	0.4000	0	0	0
1	0.2139	0.3139	0.0861	1.4449	0.2225	0.3432	0.0568	1.1683	0.2225	0.3649	0.0351	1.0926
2	0.1114	0.2046	0.0954	1.6314	0.1120	0.2333	0.0459	1.2441	0.0978	0.2362	0.0214	1.1539
3	0.1401	0.1559	0.0509	1.7810	0.0560	0.1214	0.0365	1.3666	0.0343	0.1032	0.0161	1.2615
4	0.0606	0.1142	0.0768	1.9456	0.0329	0.0699	0.0226	1.5121	0.0166	0.0428	0.0076	1.4004
5	0.0231	0.0833	0.0540	2.1934	0.0202	0.0427	0.0128	1.6834	0.0166	0.0214	0.0028	1.5213
6	0.0148	0.0548	0.0223	2.5659	0.0116	0.0274	0.0056	1.7994	0.0081	0.0173	0.0021	1.4835
7	0	0.0371	0.0000	2.9993	0	0.0172	0.0000	1.7772	0	0.0102	0.0000	1.5292
Panel B: c=.10												
	V=36.24				V=9.65				V=4.64			
	Overall average price = 1.78				Overall average price = 1.30				Overall average price = 1.18			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.3000	0	0	0	0.4000	0	0	0	0.4000	0	0	0
1	0.2184	0.2891	0.0109	1.4784	0.1932	0.3617	0.0383	1.1570	0.2018	0.3783	0.0217	1.0883
2	0.2029	0.2208	0.0085	1.5965	0.1277	0.2230	0.0084	1.2637	0.0948	0.2121	0.0113	1.1827
3	0.0814	0.1668	0.0446	1.7278	0.0615	0.1235	0.0125	1.3730	0.0430	0.0984	0.0077	1.2892
4	0.0713	0.1115	0.0145	1.9666	0.0394	0.0649	0.0092	1.5363	0.0187	0.0465	0.0041	1.4186
5	0.0606	0.0799	0.0058	2.2172	0.0279	0.0428	0.0058	1.6992	0.0127	0.0210	0.0018	1.5319

6	0.0292	0.0546	0.0119	2.5602	0.0109	0.0290	0.0047	1.7340	0.0111	0.0136	0.0009	1.5387
7	0	0.0411	0.0000	2.9048	0	0.0156	0.0000	1.7904	0	0.0120	0.0000	1.5192
Panel C: c=.20												
	V=35.46				V=9.12				V=4.36			
	Overall average price = 1.78				Overall average price = 1.31				Overall average price = 1.19			
t	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice	avgx	avgX	avgI	avgprice
0	0.3000	0	0	0	0.3500	0	0	0	0.3500	0	0	0
1	0.2274	0.2984	0.0016	1.4622	0.2247	0.3451	0.0049	1.1666	0.2175	0.3480	0.0020	1.0989
2	0.1528	0.2268	0.0021	1.5829	0.1270	0.2269	0.0028	1.2653	0.1184	0.2185	0.0010	1.1879
3	0.1051	0.1541	0.0009	1.7785	0.0665	0.1282	0.0016	1.3804	0.0448	0.1182	0.0012	1.2683
4	0.0800	0.1052	0.0008	2.0014	0.0476	0.0662	0.0019	1.5329	0.0197	0.0449	0.0011	1.4491
5	0.0613	0.0791	0.0018	2.2247	0.0319	0.0462	0.0033	1.6367	0.0099	0.0204	0.0003	1.5133
6	0.0423	0.0578	0.0053	2.5067	0.0179	0.0325	0.0028	1.7852	0.0129	0.0101	0.0001	1.5580
7	0	0.0476	0.0000	2.7325	0	0.0206	0.0000	1.7566	0	0.0131	0.0000	1.4542

The developer chooses X_t, the percentage of lots sold out of existing inventory. I_t is the percentage of lots held in inventory. The developer also chooses, x_t, the percentage of available land developed. Average values are expected values over the theta nodes at each time

Table 10 summarizes the results of all of the models estimated above in a concise fashion by dividing residential real estate markets into four broad categories, including those where developers are anticipated to: 1) phase development and hold inventory, 2) phase development, but not hold inventory, 3) hold inventory, but not phase development, and 4) neither hold inventory, nor phase development. In practice, phasing and inventory decisions are expected to vary significantly from project to project, but the typology provides useful insight into the residential land development process in different types of real estate markets. The nine preceding tables serve as an illustration of how land developers might respond to economic forces via strategic phasing and inventory decisions in over 60 different environments.

Phasing and holding inventory are expected to increase the profitability of residential land development in volatile markets where cost savings are not available from economies of scale in construction, carrying costs are low, monopoly profits can be captured by restricting the supply of developed lots, and prospective buyers face uncertainty regarding the characteristics of a subdivision. One might imagine this to be the case in situations where a developer controls an attractive parcel of land in an emerging suburban market with low property taxes and high barriers to entry, but cannot take advantage of economies of scale in construction because subcontractors have little incentive to offer preferential pricing for large projects. Consumers in such a market may be deterred by the prospect of purchasing a lot in a subdivision yet to be completed, while the developer may have an incentive to reserve lots for sale in the future in anticipation of greater market demand and higher prices as risk to consumers is reduced over time. Phased development without high levels of inventory would be expected in markets with substantial carrying costs, but characteristics otherwise similar to those described above. The value of the option to phase would be further amplified in the presence of positive or negative demand momentum.

The benefits derived from phasing and holding inventory are anticipated to be much less significant in less volatile real estate markets where economies of scale in

Table 10 Market characteristics influencing optimal phasing and inventory decisions

	Inventory	No inventory
Phasing	• Low economies of scale • Low carrying costs • Significant pricing power • Volatile market demand • Strong signaling effects	• Low economies of scale • High carrying costs • Significant pricing power • Volatile market demand • Strong signaling effects • Positive or negative price momentum
No phasing	• High economies of scale • Low carrying costs • Significant pricing power • Volatile market demand • Strong signaling effects	• High economies of scale • High carrying costs • Minimal pricing power • Limited demand volatility • Weak signaling effects

construction exist, carrying costs are high, pricing power is limited, and consumers face less uncertainty when purchasing a lot before a subdivision is completed. This may be the case in markets experiencing moderate growth, with high property taxes and few barriers to entry. Preferential construction pricing for large projects, additional demand stimulated by signaling effects, and market competition all encourage simultaneous development in this scenario, while stable market conditions and high carrying costs discourage the holding of inventory. Lower anticipated carrying costs could encourage inventory without phasing in this type of environment, although presumably not at high levels since the value of the option to sell lots in the future is low in markets with little volatility in demand.

Since individual development projects are responsive to unique combinations of the economic variables discussed throughout this paper, it is difficult to identify examples that fit neatly into each of the quadrants presented in Table 10. However, regional variations in residential development patterns support the general theoretical framework. A comparison of two stylized scenarios helps illustrate the point.

First, consider suburban submarkets throughout the southeastern and southwestern United States with an abundance of land and robust housing demand during normal economic times. Pricing power is presumed to exist in this type of environment because average homebuilder size and market concentration have been found to increase with market activity and the availability of large tracts of developable land (Somerville 1999). Demand volatility is anticipated to be relatively high in this scenario, while economies of scale in construction are expected to exist to a point.[14] Signaling effects are additionally anticipated to be present to reflect the uncertainty buyers face when purchasing in an undeveloped area that does not yet have a defined character.[15]

In the scenario described above, phasing may occur, but with a large initial investment to take advantage of economies of scale in construction. Inventory is held because of low carrying costs and sales are made evenly over time to take advantage of the pricing power and to increase prices because of positive signaling as the development is sold out. This may help explain significant inventories of developed lots in markets such as Atlanta, Dallas, Las Vegas, Orlando and Phoenix.

Contrast the environment described above with the land constrained markets of the northeastern United States where significant competition, historically volatile housing demand and relatively high property tax rates exist. Pricing power is presumed to be low, as are economies of scale in construction because of competition among numerous small residential land developers. Carrying costs are

[14] Although most carrying costs are not expected to vary dramatically across geographic areas, property taxes in some jurisdictions are considerably higher than others. The U.S. Census American Community Survey (2008), for example, identifies property taxes in several counties surrounding Chicago, New York and San Francisco with property taxes 300% to 600% higher than the national average. The cost of holding undeveloped land for extended period of times may be extremely costly in these parts of the country, as compared to many parts of the Midwest or Southern United States.

[15] Galster (2001) noted that consumer preferences for homogenous neighborhoods, coupled with restrictive land use regulations, limits diversity in the housing stock in small geographic areas. This may reduce the importance of signaling effects in areas that already have observable characteristics due to previous residential development activity. However, signaling effects may be more important to buyers considering housing on the urban fringe who cannot look to surrounding subdivisions to predict the future characteristics of the area.

high, in correspondence with high property tax rates, and signaling effects are presumed to be relatively low to reflect an area with a defined urban character resulting from numerous comparable subdivisions in close proximity. If large tracts of land are available in this type of market, synchronized phasing of development and sales will take place with little inventory held. These predicted outcomes are consistent with more conservative inventories of developed residential lots observed in markets surrounding Boston and several other northeastern cities.

Conclusions

The model presented in this paper contributes to the existing residential land development literature by using a real options framework to evaluate the impact of several different economic variables on optimal phasing and inventory decisions concurrently. Economies of scale in construction, pricing power, inventory carrying costs, signaling effects, demand momentum and demand uncertainty are all found to have profound, and often countervailing, effects on the estimated profitability of residential land development projects when modeled in an exogenous strategic setting. Optimal production strategies are therefore expected to vary depending upon the type of project being developed, the strategic location of the site, and demand volatility in the marketplace. Prudent land developers must evaluate the unique characteristics of their projects and craft production strategies that reflect the economic environment in which they are situated in order to address these issues. The model additionally indicates that full development, smooth phased development and lumpy development patterns can all be optimal under rational decision-making, which may help explain why developers in some markets where caught with substantial inventories of residential lots heading into the financial crises, while in other markets developers were exposed primarily to risk associated with ownership of raw land.

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