Adv. Micro Theory, ECON 6202-090

Assignment 5, Fall 2010

Due: Monday November 8th

Directions: Answer each question as completely as possible. You may work in a group consisting of up to 3 members – for each group please turn in only 1 set of answers and make sure all group member names are on that set of answers. All group members will receive the same grade.

1. Producing output y requires only input x. The production function is

$$y = 70\sqrt{x}$$

   Let w denote the price of input x. Compute the marginal cost and the average cost of producing y. Verify that the average cost is less than the marginal cost for all values of y. Explain why this is so.

2. Consider a firm with the cost function

$$c(y, w_1, w_2) = y^2(w_1 + w_2),$$

   where w_i denotes the price of input i for $i = 1, 2$. Let p denote the output price. Derive the output supply function $y(p, w_1, w_2)$, and the input demand functions $x_i(p, w_1, w_2)$ for $i = 1, 2$.

3. Consider a firm with production function

$$y = (x_1^\rho + x_2^\rho)^\alpha,$$

   where $0 < \rho < 1$, and $\alpha > 0$.

   a For what value of ρ and α are there (i) increasing returns to scale; (ii) constant return to scales; (iii) decreasing returns to scale?

   b Suppose that there are decreasing return to scale. Find the long run cost function. Derive the output supply function and the input demand functions for this long-run cost function.

4. Consider a firm with a linear production function:

$$y = \sum_{i=1}^{n} \alpha_i x_i$$

   with input prices $w_1, ..., w_n$ for each of the n inputs, with each $w_i > 0$.

   a Assume that input prices are such that there are no two inputs i and j where:

$$\frac{w_i}{w_j} = \frac{\alpha_i}{\alpha_j}$$

   Find the combination of inputs that minimize the firm's cost of producing a particular amount of output y.

   b Find the cost function and conditional input demand functions for all possible values of input prices.