MONEY and BANKING: ECON 3115

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Chapter 4: Understanding Interest Rates

To simplify discussion these notes ignore important economic differences between financial intermediaries and financial markets. Instead, the notes treat the financial system as one big happy family.

Outline

I) What is the 'Present Value' of an Asset? How is Present Value Calculated?
II) Interest Rates and *Yield to Maturity*
III) Interest Rates versus *Rates of Return*
IV) Interest Rate Risk
V) "The" Interest Rate and Economic Efficiency
VI) The Real Interest Rate versus the Nominal Interest Rate

I) What is the 'Present Value' of an Asset? How is Present Value Calculated?

1) Present Value (*PV*) is a fundamental concept in financial economics. For this reason, you need to understand the concept and how to calculate it. Start with the concept. Generally speaking, the *PV* of an asset is the current value of future income the asset is expected to produce, after the future income is discounted by the "opportunity cost of owning the asset." The opportunity cost of owning an asset is the future income that could have been earned if the owner had bought the next best alternative asset. E.G., suppose Ms. Penny-pincher ranks four different assets with the colorful names Red, Blue, Green, and Fuchsia. Ms. P. ranks Blue highest, and buys only Blue. She ranks Green second highest, but does not buy Green. Thus, Ms. P. foregoes the opportunity to earn income from Green. The foregone Green income is the opportunity cost of owning Blue.

2) The opportunity cost of owning an asset often is called the "time value of money" because it is the value one could have achieved, over time, had "money" been invested differently. So you will sometimes see *PV* described as the expected future income from an alternative asset after adjustment for the time value of money.

3) Interest rates are the rates of return on 'debt instruments,' such as bonds, loans, etc. 'Debt instrument' is an awkward term, so use the word 'bond' as shorthand for all debt instruments and loans, hereafter. In financial economics, the *PV* of a bond is the highest price a rational saver would be willing to pay for the bond. To see why, consider yourself as an example. You are planning your saving program for the next year. You have two mutually exclusive saving opportunities, deposit your savings, risk-free, in a bank for the year, or use the savings to buy a risk-free bond that matures in a year. What is the highest price you should be willing to pay for the bond? Because risk is not an issue, you need to know only the *PV* of the bond, as described next.

A) The highest price you should be willing to pay for the bond depends on the future income you expect the bond to produce, and, of course, the bond's opportunity cost. The opportunity cost is the interest income on the next best opportunity that must be foregone, if you were to buy the bond. In the current case, the opportunity cost of owning the bond is the interest income that could have been earned if you were to deposit your savings in a bank.

B) Therefore, at most you should be willing to pay a price such that you would then expect the bond to earn ***at least*** its opportunity cost. To calculate the numerical value of *PV* use the interest rate on the bank deposit and the expected future income, or Cash Flow (*CF*), from owning the bond.

C) To focus on the heart of the problem, use the following simplifying assumptions: a) There is no default risk: That is, if you buy the bond you know you will receive *CF* dollars in one year, with absolute certainty. b) Income tax rates on the bond's *CF* and interest on bank deposits are the same. c) Financial markets are perfectly competitive, so the market price of bonds always equals *PV*.

D) "Market interest rate" is a synonym for opportunity cost, and is more common, so I will use the former term hereafter.

E) You should not be willing to buy the bond unless its *CF* generates income at least equal to the market interest rate. Let *i* stand for the market interest rate. *CF* must equal the price you pay today, which equals *PV*, plus what you could earn in the market, which equals $i \cdot PV$. Therefore,

Equation I) $$CF = PV + i \cdot PV = PV(1+i)$$

Solving this for *PV* gives

Equation II) $$PV = \frac{CF}{1+i}$$

Equation II) says the *PV* of the bond is *CF* “discounted” back to the present at rate *i*. The term 'discounted' is used here because dividing *CF* by the gross market interest rate (1+*i*) causes the Present Value to be less than *CF*.

F) Many types of bonds mature two, three, or any number of years in the future. Equation II) can be generalized for such cases. E.G., if the market interest rate is constant from year to year, the *PV* of a bond paying *CF* *n* years in the future is [1]

Equation III) $$PV = \frac{CF_n}{(1+i)^n}$$

4) Equation III) indicates a ***crucial*** relationship between the bond’s the market interest

[1] Equation 3) could be re-written to account for the fact that different bonds deliver different cash flows in different time periods. This would not add much to the discussion, so we ignore it.

rate and *PV*: Ceteris paribus, the higher is *i*, the lower is *PV*.

5) But why should the price be lower if the market interest rate is higher? Mathematically, it should be obvious from Equation III) that an increase in *i*, ceteris paribus, reduces *PV*. But this is math, not economics. What are the economics of the situation? The answer is, the higher is *i*, the higher is the opportunity cost. Ceteris paribus, higher opportunity costs are undesirable. Therefore, the higher the interest rate, the less savers are willing to pay to own the bond's fixed future cash flows. It follows that higher market interest rates, ceteris paribus, cause bond prices to decline.

6) You must understand this inverse relationship between *i* and *PV*. However, you will not need to solve equations like II) and III) for this class. This is not a course in arithmetic or personal finance.

II) Interest Rates and *Yield to Maturity*

1) There are many ways to calculate the interest rate on a bond. E.G., two types of interest rates are yields on a discount basis, and coupon yields (see the textbook). However, these measures of the interest rate are not suitable in economic modeling of the financial system because they do not account for a bond's opportunity cost. Generally speaking, *PV* of ***any*** asset is based on the asset's opportunity cost, so the economically relevant calculation of the interest rate must be based on *PV*. The "Yield to Maturity" (*YTM*) is calculated from a bond's *PV*, so economists favor the use of *YTM* to measure the interest rate on bonds.

2) Here's the definition:

YTM is the interest rate on a bond calculated by equating the PV of the bond's future cash flows to the bond's current price.

3) Equation II) shows how to calculate the *PV* of a bond with a single future cash flow received one year after purchase. What is the *YTM* on such a bond? Let P_t be the market price of the bond when it is purchased. Let CF_{t+1} be the cash flow expected after one year. Because *YTM* is the interest rate found by equating the *PV* of CF_{t+1} with P_t, *YTM* can be derived by solving the following equation:

Equation IV) $$P_t = \frac{CF_{t+1}}{1+i}$$

The right-hand side of IV) is the bond's *PV*. The left-hand side is the current price. Therefore, *i* in the denominator of the right-hand side must, by definition, be *YTM*. So to get YTM, simply solve IV) for *i*. This gives

Equation V) $$YTM \equiv i = \frac{CF_{t+1} - P_t}{P_t}$$

Equation V) can be written more simply as

Equation V") $YTM = \frac{CF_{t+1}}{P_t} - 1$

Note the inverse relationship between *i* and P_t in equation V"). If P_t rises, ceteris paribus, *i* must decline. I.E., the more you pay for a bond today, everything else constant, the lower is its interest rate and *YTM*. Earlier we saw that the higher the interest rate, ceteris paribus, the lower is *PV*. Here we see that the higher the price paid for the bond, the lower is *YTM.* These are just two ways of saying the same thing: In general, there is an inverse relationship between asset prices and interest rates.

III) Interest Rates versus *Rates of Return*

1) We must distinguish between the concepts "interest rate" and "rate of return." These sound an awful lot alike. But only under restrictive circumstances will the interest rate on a bond will be equal its rate of return. More often rates of return diverge by large amounts from interest rates.

2) *Time* is the trait that differentiates interest rates on bonds from their rates of return:

A) Interest rates look forward from the present into the future, so they are calculated based on future cash flows. This forward-looking perspective of the interest rate is often described by saying the interest rate is calculated *ex ante*, that is, "before the fact," meaning before cash flows are earned. The interest rate is the income, in percent, that would be earned on a bond from today into the ***future***, if market conditions do not change before the bond is sold.

B) Rates of return reverse the time perspective a full one hundred and eighty degrees. Rates of return are retrospective, or backward-looking, so we say the rate of return on an asset is calculated *ex post*, that is, "after the fact," meaning after cash flows have been earned. This is true even if the return has not been "realized," that is, even if the owner has not sold the bond.

3) So what is the restriction that must hold for a bond's interest rate to equal its rate of return? The two rates will be equal only if the bond is held to maturity. More on this below.

IV) Interest Rate Risk

1) It is easy to find out what the current interest rate on bond is. E.G., the Money and Investing section of the *Wall Street Journal* publishes the current *YTM* on government bonds every work day. Unfortunately, no one, not even Warren Buffett, can be certain about what the market interest rate will be in the future. If, after a saver buys a bond, market interest rates rise unexpectedly, bond prices will decline unexpectedly. So the possibility that future market interest rates will rise unexpectedly creates a risk to bond owners. Thus, "interest rate risk" is defined as *the risk to bond owners of an unexpected increase in future market interest rates*.

2) The relationship between a bond's term to maturity and its interest rate risk.

A) Consider the following unique case in which interest rate risk is zero. First, assume the bond has zero default risk: I.E., the borrower will repay the bond at maturity with 100% certainty. Second, assume the bond's *YTM* on the day it initially is issued is **10%**. And third, assume Mr. Penny-pincher buys the bond, and does not sell it until the day it matures.

B) What rate of return will Mr. P. have earned when the bond matures? The answer must be **10%**. Default free bonds held to maturity always deliver a *rate of return*, ex post, equal to the interest rate that was calculated, ex ante, on the day the bond initially was issued. After all, ex ante, *YTM* was calculated on the assumption that the bond will pay the promised cash flows at maturity. If, ex post, the borrower delivers those promised cash flows, the saver will have earned what he expected, namely *YTM*.

C) Now consider default free bonds that possess interest rate risk. In order to have interest rate risk, bonds must be sold before they mature. Suppose today Mr. P. buys a U.S. Treasury bond with five years to maturity. She plans to sell the bond after only a year, well before maturity. Of course, Ms. P. knows *YTM* today. She hopes that the eventual rate of return on the bond, one year from now, will be equal to or greater than *YTM*. However, she cannot be certain, because she cannot know what market interest rates will be when she sells the bond in one year. Suppose that before one year is up, the market interest rate rises: The bond's market price must decline, so the rate of return will fall below the initial *YTM*. Thus, the possibility of an unexpected increase in the market interest rate creates interest rate risk on bonds sold before they mature.

D) How does the possibility of unexpected increases in future market interest rates affect the rate of return on bonds *held to maturity*? It doesn't. If a default free bond is held to maturity, the bond's rate of return will equal its initial *YTM*, no matter how market interest rates change. Think about it.

3) The discussion so far explains why default free bonds suffer from interest rate risk: bonds sold before they mature suffer capital loss if the market interest rate rises unexpectedly. But this does not explain why some bonds have more interest rate risk than others. Term to maturity is the difference between different bonds' interest rate risks:

General Principle: The longer the maturity of an asset, ceteris paribus, the larger is the asset's interest rate risk.

As you already have learned, the longer is a bond's maturity, the larger is its opportunity cost. In addition, the longer is a bond's maturity, the larger must be ***increases*** in opportunity cost resulting from an ***increase*** in the market interest rate. And the larger the ***increase*** in the opportunity cost, the larger the resulting decline in the bond's price.

A) Here's an example: Consider two bonds with different maturities but with initially equal *PVs*. If the market interest rate is 5%, a bond delivering cash flow of \$100 in one year and another bond delivering cash flow of \$121.55 in five years both have *PVs* equal to \$95.24. I.E., $\frac{\$100}{1.05}$ and $\frac{\$121.55}{(1.05)^5}$ both are approximately equal to $\$95.24$. Nevertheless, if the market interest rate increases from 5% to 10%, *PV* of the one year bond declines to \$90.90, while *PV* of the five year bond declines to \$75.47. The increase in the market interest rate causes the longer maturity bond's value to decline more. This is true of all assets. Thus, the principle: *the longer the maturity of an asset, ceteris paribus, the larger is the asset's interest rate risk.*

B) This principle makes economic sense. The *initial* (before market rates increase) opportunity cost of a longer maturity asset is larger than that of a shorter maturity asset. If the market rate *increases*, the longer maturity asset suffers from the *increase* in opportunity cost for a longer period of time, so that asset loses more value.

C) It is important to keep in mind that this principle has nothing at all to do with default risk, which is another matter entirely. The principle shows that even default-free bonds are risky, if they are sold before they mature.

V) “The” Interest Rate and (here we go again) Economic Efficiency

1) Each day consumers and firms pay and receive a huge number of different interest rates. Consumers receive interest on bank deposits, CDs, and government and corporate bonds. Consumers pay interest on mortgages, car loans, and credit cards. Firms pay and receive an equally diverse set of interest rates. Nevertheless, in Money and Banking and in Macroeconomics we can ignore the immense diversity of interest rates and speak as if there is only one, representative, market interest rate, namely "the" market interest rate.

2) It becomes tiresome to repeat the term "market" over and over, so we will drop it. Thus, you should read 'the interest rate' to stand for 'the market rate of interest,' hereafter.

3) But what justifies ignoring all the real-life diversity in interest rates? Interest rates on different bonds tend to rise and fall together over time. In statistical terms, interest rates on different bonds are highly correlated. This is evident in Figure 1 on page 4 in the textbook, which graphs interest rates on three-month Treasury bills, long term federal government bonds, and corporate bonds, between 1950 and 2008. Notice how the interest rates seem to rise and fall together. The correlation coefficient of the interest rates on 1 year and 10 year government debt in Figure 1 is a very high 0.93. Why are interest rates highly correlated?

A) Different bonds tend to be close substitutes. If the interest rate on one type of bond, say U.S. Treasuries, increases, ceteris paribus, some savers will substitute Treasuries for other bonds, say corporates. The substitution away from corporates is a decline in demand, which drives down their price, driving up the corporate interest rate. Thus, the interest rate on corporates rises in sympathy with the interest rate on Treasuries.

B) If the interest rates on two different assets are completely synchronized, rising exactly at the same time and falling exactly at the same time, we say the interest rates are perfectly correlated.[2] Although not perfect, the correlation between different interest rates in the economy is high enough that for many important economic issues, we can treat all bonds as a single asset. This permits us to speak as though there is a single representative interest rate that can stand in for the many diverse actual interest rates in the economy. We refer to this representative interest rate as "the" interest rate.

4) What if *the* interest rate is high? Is that efficient or inefficient? The correct answer is far from obvious. The information contained in economic statistics often is subtle and counter-intuitive. It takes training and a nose for detail to understand the message hidden in the entrails of economic statistics. E.G. sometimes economic inefficiency causes the interest rate to be high, but other times efficiency causes the high interest rate to be high!

A) This sort of ambiguity is very, very common in economics. It explains why two people can look as the same data and draw opposite conclusions. E.G., economists enjoy correcting a common newspaper mistake by pointing out that an increase in the unemployment rate does not always signal deterioration in labor markets. The reason is that when the economy begins recovery from a recession, an increase in worker confidence tends to draw discouraged workers into the labor market, increasing the supply of labor, and driving up the unemployment rate. An increase in the unemployment rate signals labor market deterioration only if it is caused by a decline in demand for labor. Likewise, a high interest rate can be a signal of economic improvement or economic malaise: one cannot know without understanding the cause of the high interest rate.

B) An increase in the interest rate may signal inefficiency. Asymmetric information causes risk in the transfer of funds from savers to borrowers. When savers incur risk, they require compensation, in the form of elevated interest rates. But an increase in the interest rate causes some safe borrowers with productive investment opportunities to drop out of the market. In periods of rising asymmetric information, the interest rate rises, and socially valuable transfers fail to take place, making finance more inefficient. In this case the high interest rate is a signal of greater inefficiency.

C) However, an decrease in inefficiency can cause the interest rate to increase! E.G., an increase in the productivity of capital causes firms to increase their demand for capital. Firms finance capital by borrowing, that is, by selling bonds. The increased supply of bonds drives down their price, driving up the interest rate

[2] Technically speaking, returns on two assets are perfectly correlated if when the return on one asset is above its mean, the return on the other also is above its mean; and when the return on one asset is below its mean, the return on the other also is below its mean.

(we model this in Chapter 5). Although the increase in productivity causes the interest rate to rise, the economy is less inefficient. In this case, the high interest rate is a signal of lower inefficiency.

D) We should not automatically assume high or increasing interest rates are signs of economic deterioration. As often occurs in economics, whether higher prices signal higher or lower economic welfare depends on the source.

VI) The Real Interest Rate versus The Nominal Interest Rate

1) Even after all the work done above, we still got a problem. After all, the interest rate described so far is not adjusted for inflation! This is not good. The inflation-unadjusted interest rate is called the "nominal" interest rate. Think of 'Nominal' as meaning "in name only," or not "real." But real, not nominal, prices are the drivers of economic decisions. Why?

2) Ceteris paribus, inflation reduces the real interest rate. To see this, let π be the inflation rate, let i be the nominal interest rate, and let i_r be the real interest rate. If i=5%, and π=2%, then $i_r = i - \pi = 5\% - 2\% = 3\%$. And if π increases to, say, 5%, then $i_r = 5\% - 5\% = 0\%$. In this case, after bond holders are paid, the real rate of return is zero. Bond holders have earned nothing for their efforts.

3) Most lenders and most borrowers understand that an increase in inflation, ceteris paribus, reduces the real interest rate**.** Irving Fisher, perhaps the most influential and important monetary economist of all time, explained how this knowledge affects markets and causes the nominal interest rate to change. Fisher conjectured that if bond market participants expectation of future inflation increases, they will adjust the nominal market interest rate higher, to compensate for the increase. If so, the market effectively compensates lenders for expected inflation by building an expected inflation premium into the nominal interest rate. In this case, the nominal interest will be equal to the sum of the real interest rate and the expected inflation rate. This idea is expressed mathematically in the famous Fisher equation:

Fisher equation) $$i = i_r + \pi^e ,$$

where π^e is the expected inflation rate. The Fisher equation implies that if bond owners' real required rate of return is 7%, but they expect inflation to be 2%, the market will set the nominal interest rate to 9%. In this case, bond owners will be compensated for the inflation they expect because, ex post, at the end of their holding period, the real rate of return they earn will be $i_r = i - \pi = 9\%\text{-}2\% = 7\%$, which is what they require.

4) In class, we will talk about empirical evidence for and against the Fisher equation.