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VOLUNTARY DISCLOSURE OF NEGATIVE INFORMATION AND ITS EFFECT ON COMPETITION

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ABSTRACT

Contrary to conventional wisdom that sharing negative information hurts, many sellers voluntarily disclose weaknesses of their products in markets where information asymmetry is high. This paper provides an explanation on this type of seller's honesty by developing a framework where sellers with low-quality products can benefit from a disclosure of negative information. We consider a lemon-market model where the product's quality is unobservable by buyers and buyers dislike the risk associated with purchasing the product of uncertain quality. Sellers can send a cheap-talk message indicating whether their quality is high or low. We describe equilibria where, with positive probability, low-quality sellers voluntarily disclose negative information about their product. The disclosure is beneficial for sellers as it results in product differentiation: there is no quality uncertainty about products labeled as low quality, while there is uncertainty about products labeled as high-quality.

Voluntary Disclosure of Negative Information and Its Effect on Competition*

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Abstract

Contrary to conventional wisdom that sharing negative information hurts, many sellers voluntarily disclose weaknesses of their products in markets where information asymmetry is high. This paper provides an explanation on this type of seller's honesty by developing a framework where sellers with low-quality products can benefit from a disclosure of negative information. We consider a lemon-market model where the product's quality is unobservable by buyers and buyers dislike the risk associated with purchasing the product of uncertain quality. Sellers can send a cheap-talk message indicating whether their quality is high or low. We describe equilibria where, with positive probability, low-quality sellers voluntarily disclose negative information about their product. The disclosure is beneficial for sellers as it results in product differentiation: there is no quality uncertainty about products labeled as low quality, while there is uncertainty about products labeled as high-quality.

1 Introduction

Is honesty the best policy for sellers? The well-known maxim does not seem to be embraced by firms, who often choose to conceal negative information about their products and services, especially when there exists information asymmetry between firms and their customers. The U.S. Senate's report on the global financial crisis of 2008 (Levin and Coburn, 2011) documents many cases where financial institutions hid negative aspects of their products, which, according to the report, was one of the main causes of the crisis. Frankel et al. (2010) found that 69 percent of imported olive oils labeled "*extra virgin*" do not actually meet the standard. Similar examples of incomplete disclosure under informational asymmetry can be readily observed in many other

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markets where the information asymmetry is high. Marketing literature has also supported sellers' unwillingness to reveal negative information by documenting how negative information damages sellers' performance. According to those studies, negative information decreases sales and purchase likelihood through various routes such as publicity (Tybout et al., 1981; Wyatt and Badger, 1984), word-of-mouth (Arndt, 1967; Engel et al., 1969; Haywood, 1989; Laczniak et al., 2001; Mizerski 1982; Wright 1974), and customer reviews (Basuroy et al., 2003; Chevalier and Mayzlin 2006; Clemons et al., 2006; Dellarocas et al., 2007; Reinstein and Snyder 2005).

What we find interesting is that it is also not uncommon to witness sellers voluntarily sharing negative aspects of their products and services and claiming low quality. Many online retailers, such as Amazon.com, voluntarily disclose weaknesses about the products they offer through various routes such as customer reviews on their own websites. Woot.com is especially famous for its preemptive revelation of the disadvantages of listed products. On their website they state that they would prefer customers not buying from them to regretting their purchases.[1] Hans Brinker Hotel in Amsterdam, Netherlands is famous for its strategy of honestly revealing its low quality to customers, and actively explaining negative aspects of their services, such as rooms without a view and no hot water. Nevertheless, many travelers visiting Amsterdam choose to stay at this hotel and leave positive reviews such as "*For the reputation of the world's worst hotel, it wasn't as bad as I thought.*"[2] In the olive oil example mentioned above, some sellers do label their product as either "*virgin*" or "*lampante virgin*" which are inferior grades to "*extra virgin.*" This is despite the fact that it is possible for sellers to overstate the quality, as most customers cannot tell the difference.

Although we can easily observe instances where sellers voluntarily reveal inferior information about their products, this phenomenon has not received much attention in academic literature. This paper aims to fill this gap.

In our paper we develop a model where there are two sellers whose products are of either low or high quality. Buyers cannot observe the quality of the product, but all prefer high quality over low quality.[3] Buyers dislike uncertainty regarding the quality of the product. We use two alternative frameworks to model this uncertainty-aversion. The first framework is a perceived risk framework used in consumer behavior literature (see Dowling, 1986; Srinivasan and Ratchford, 1991). If the purchased product has a quality lower than the buyers expected, they experience disutility proportional to the difference between expected and actual qualities.[4] The second framework is the standard expected-utility framework. We will refer to buyers' risk attitude under the perceived risk framework as "*risk-sensitivity*," and under the expected utility framework as "*risk-aversion.*" We will use a generic term"*uncertainty-aversion*" when talking about both frameworks. Buyers differ

[1] http://www.woot.com/faq?ref=ft_wiw_faq (accessed on November 14, 2015). Archived version: https://archive.is/b9cxI

[2] http://abcnews.go.com/Travel/proud-worlds-worst-hotels/story?id=17696356 (accessed on November 14, 2015). Archived version: https://archive.is/N9ZhL

[3] This is different from Tadelis and Zettelmeyer (2015) where some buyers prefer lower quality, while others prefer higher quality.

[4] This definition of perceived risk is similar to the concept of disappointment-aversion in decision-making literature (see e.g. Bell, 1985, and Loomes and Sugden, 1986).

in the strength of their uncertainty-aversion. Other things being equal, buyers with high (low) uncertainty-aversion are more (less) likely to purchase the product with low uncertainty.

There are three stages in the game. First, the sellers, who observe the quality of their product, send one of the two messages: L (low quality, e.g. "*virgin oil*"), or H (high quality, e.g. "*extra virgin oil*"). Messages are cheap talk in that there is no cost associated with sending either message. Second, sellers observe each other's messages and simultaneously determine prices for their products. Third, the nature draws a buyer's degree of uncertainty-aversion and the buyer chooses a seller from whom to purchase the product.

We are interested in the equilibria where low quality sellers, i.e. those who have negative information to disclose, send message L with a positive probability, and high-quality sellers send message H with probability 1. In these equilibria, the claim that the product is of high quality can be false, as its true quality might be either high or low, and the claim that the product is of low quality is always true. This parallels what we observe in most markets with information asymmetry, as it is low-quality sellers who may choose to misrepresent themselves while high-quality sellers seldom claim to be low quality. The benefit of disclosing the negative information comes from the fact that it introduces product differentiation into the market. If neither seller discloses negative information, either because they are high types or because they choose not to, then from buyer's point of view their products are identical (see also footnote 6) and the only factor that matters is the product's price. In the asymmetric case, where only one seller discloses negative information, a buyer's trade-off changes. The product with negative information is, on the one hand, less valuable due to its revealed low quality but, on the other hand, there is no risk associated with the purchase as the quality is known. The product without negative information is, on the one hand, more valuable due to the presence of high-quality sellers, but, on the other hand, the quality of the product is uncertain. Metaphorically speaking, it becomes a choice between a known devil and an unknown angel. Buyers with high degree of uncertainty-aversion prefer a known devil, i.e. the product with disclosed negative information. Buyers with low degree of uncertainty-aversion prefer an unknown angel, i.e. the product without negative information. This creates product differentiation, softens the competition, and increases profits of both high- and low-quality sellers.

We investigate conditions necessary for an equilibrium with negative information to exist. We derive three conditions. First, the share of high-quality sellers cannot be too large. Otherwise, the risk associated with purchasing the product with uncertain quality is too low and then the low-quality sellers find it optimal to mimic high-quality sellers rather than to reveal their type. Second, distribution of buyers' uncertainty-aversion must have sufficiently large support. While intuitive — buyers with high degree of uncertainty-aversion are those who are more likely to purchase product with a lower but certain quality — this condition is sufficient in the case of perceived-risk framework but not in the expected utility framework. In the latter case, if buyers' risk-aversion is, for example, uniformly distributed, the equilibrium with negative information does not exist regardless of how large its support is. Finally, and somewhat counterintuitively, for an equilibrium with negative information to exist there must be a sufficiently large number of buyers with *low* degree of uncertainty-aversion. The intuition is as follows. Sellers who disclose negative information serve buyers with high uncertainty-aversion, while sellers who do not disclose serve buyers with low uncertainty-aversion. Having sufficiently many buyers who are not too risk-sensitive (risk-averse) makes sellers who do not disclose negative information less interested in competing for buyers with high risk-sensitivity (risk-aversion), thereby creating a niche for sellers who choose to disclose negative information.

In the case of uniform distribution we study how the equilibrium changes with parameters. First, as the quality difference between low- and high-quality products increases, it makes the equilibrium with the disclosure of negative information more profitable for both high- and low- quality sellers. Second, if there are more risk-sensitive customers in the population, it makes disclosure of negative information more likely and the market share of low-quality sellers who disclose negative information goes up. Finally, the share of high-quality products on the market has non-monotone impact. When it is too high or too low then the risk associated with purchasing the H-product, e.g. oil labeled as "extra virgin", is low, reducing the benefits of negative information disclosure and product differentiation. The product differentiation is the highest for intermediate values of the share of high-quality products.

Overall, this study belongs to a limited group of papers that aim to re-examine the conventional wisdom about the disclosure of negative information in the presence of information asymmetry. We develop model with information asymmetry where low-quality sellers have incentives to reveal their quality. A novel aspect of our paper is that quality disclosure comes solely from low-quality sellers.

The paper is organized as follows. In Section 2 we review review related literature. In Section 3 we introduce our model using uniform distribution and link it to the Hotelling model of horizontal differentiation. In Section 4 we develop the model in general case and solve for equilibrium. All proofs are in the appendix.

2 Literature Review

This study belongs to a large body of literature studying markets in the presence of information asymmetry. Akerlof (1970) demonstrated the possibility of market failures caused by information asymmetry, which spurred numerous theoretical studies explaining how verifiable information disclosure can mitigate adverse selections under information asymmetry (e.g., Grossman & Hart, 1980; Grossman, 1981; Milgrom, 1981; Jovanovic, 1982; Viscusi, 1978).[5] These studies have argued that full disclosure naturally happens in markets with information asymmetry for the following reason. If a seller does not disclose the quality, then a buyer will assume that the quality is the lowest in the set of possible quality levels, since the seller could have made more profit by disclosing true quality if its quality is not the lowest in the subset. Therefore, the disclosure of information happens top down in an "unraveling process," in such a way that full disclosure starts from the player with the highest quality and goes down to players with lower quality. This unraveling argument is based on the assumption that lying is not possible either because it is prohibited by law (Grossman & Hart,

[5]See, Dranove and Jin (2010) for a detailed literature review on information disclosure.

1980; Grossman, 1981; Milgrom, 1981) or ex post verification is readily available (Jovanovic, 1982). Our paper differs from unraveling literature in that we assume that sellers can misrepresent their type and that quality disclosure comes from low-quality sellers.

This study also contributes to an understanding of the effect of information disclosure on competition. Board (2009) has shown that a firm may not disclose information in a competitive environment, as disclosing information may lead to stronger competition. Therefore, when one high-quality firm discloses, other firms compare the cost of increased competition if they disclose with the cost of reduced perceived quality if they do not disclose. Cheong and Kim (2004) have argued that no firm will disclose information as the number of competing firms becomes infinity and when the market is almost perfectly competitive, since the benefit from information disclosure gets smaller due to price competition. Hotz and Xiao (2013) have focused on the heterogeneity in the market in terms of product attributes and customer preferences and have shown that neither high-quality sellers nor low-quality sellers would disclose information, as more information results in more elastic demand, which in turn leads to increased price competition among the firms. On the other hand, Jin (2005) has observed health maintenance organizations' (HMOs) voluntary disclosure of product quality and found that the disclosure decision differs depending on the level of competition. She has presented that disclosure decision is driven by incentives to differentiate from competitors and that disclosures are more likely to operate in highly competitive markets. This empirical results are similar to our paper where quality disclosure generate product differentiation.

The incentive for sharing negative information in markets has been relatively under-studied in the literature while numerous studies have explored the incentive for sharing positive information. The marketing literature, as mentioned in the introduction, has generally shown how negative information might damage sellers' performances (Tybout et al., 1981; Wyatt and Badger, 1984; Arndt, 1967; Engel et al., 1969; Haywood, 1989; Laczniak et al., 2001; Mizerski 1982; Wright 1974; Basuroy et al., 2003; Chevalier and Mayzlin 2006; Clemons et al., 2006; Dellarocas et al., 2007; Reinstein and Snyder 2005). The economics literature has also agreed that there exist incentives for sellers to reveal high-quality. In his seminal paper, Spence (1973) has shown that it is high-quality employees who will try to separate themselves from low-quality employees through higher education. Milgrom (1981) has called it a monotonicity property that higher-quality types always try to signal or disclose information more than lower-quality types, as disclosing the information about low quality will result in lower valuation than no disclosure. Akerlof (1976) has shown how more talented workers try to work faster than the socially optimal pace to differentiate themselves from less talented workers. At the same time, there are a few recent papers that have analyzed how sharing negative information can help sellers. Berger, Sorensen, and Rasmussen (2010) have shown the circumstances where negative publicity can benefit sellers. In their study, they have found that negative reviews in the *New York Times* actually raised awareness of some less-known books, resulting in the increased sales. Tadelis and Zettelmeyer (2015) have found that negative information can act as a matching mechanism and increase sales for low quality products when there exist separate markets for products with different quality levels. Our paper differs from Tadelis and Zettelmeyer in that in our model all buyers prefer high quality over low quality. The information disclosure serves not as a matching mechanism but as a way to differentiate the product from competitors.

3 Model Setup and Uniform Distribution Example

3.1 The model setup and timing

The supply side of the market is built upon a standard duopoly model with vertical differentiation (see e.g. Shaked and Sutton, 1982; Board, 2009). There are two sellers and each seller has one unit of a product. The quality of each product is exogenous, and it is private information that is observed by the seller only. The product's quality can be either high or low, and the probability of the high quality product is q. Each seller, regardless of his product's quality, can send one of two messages: L, indicating that the product quality is low; or H indicating that the product quality is high. For example, H message can be labeling the product as "*extra virgin oil*" and L message can be labeling the product as "*virgin oil*," which is an inferior grade. Messages are cheap talk in that both messages are costless to send. Given the focus of the paper, we will be interested in equilibria where only (some) low-quality sellers can choose to send message L, while high-quality sellers would always send message H. This parallels what we observe in actual market with information asymmetry, where it is virtually unheard of for high-quality sellers to pretend to be low-quality.

An alternative interpretation can arise when credible disclosure of negative information is possible and is costless while credible proof of the product of being high quality is prohibitively expensive. Then low-quality sellers can choose whether to reveal negative information, i.e. send message L, or not, i.e. send message H. High-quality sellers have no choice but to send message H, as they do not have any negative information to disclose.

Buyers value quality i as v_i, where $i \in \{L, H\}$. Buyers prefer higher quality, $v_H > v_L$, but dislike having uncertainty about the product's quality. We model buyer's attitude towards the uncertainty using two alternative frameworks. The first one is the perceived-risk framework where perceived risk is defined as the probability of loss times the size of the loss from a purchase (Dowling, 1986; Srinivasan and Ratchford, 1991). When a buyer purchases a product of expected quality Ev, pays price p, and the product's actual quality is v the buyers' utility is

$$U_b^{PR} = Ev - p + b \cdot E[v - Ev | v \leq Ev]. \tag{1}$$

The last term in U_b^{PR} is perceived risk associated with the purchase. When the quality of the purchased product, v, is less than the expected quality, Ev, then the buyer experiences loss of size $v - Ev$. Conditional on experiencing loss, $v < Ev$, the average perceived risk is $E(v - EV | v \leq EV) < 0$. Parameter $b \geq 0$ measures the degree of buyer's sensitivity towards perceived risk. We assume that b is distributed with cdf $\Phi(b)$. We further assume that $\Phi(b)$ has a positive differentiable density $\phi(b)$ and its support is $[0, B]$, where B can be infinity.

The second framework is the expected utility framework. Buyers have concave Bernoulli utility function, $u(\cdot)$, with constant absolute risk-aversion. Buyers' utility from purchasing a product with uncertain quality v at price p is

$$U_b^{EU} = E_v u(v-p). \tag{2}$$

We denote γ as buyer's degree of absolute risk-aversion, $-u''(x)/u'(x)$. With a slight abuse of notations, we will use $\Phi(\gamma)$ to denote the distribution function of buyers' risk-aversion.

Both perceived risk and expected-utility frameworks capture the idea that buyers receive disutility when the product's quality is uncertain. The perceived-risk framework has a simpler functional form, making it more tractable. However, just like disappointment-aversion or variance-aversion frameworks, the perceived-risk preferences can violate state dominance. When b is high the buyer might prefer a product with known low quality over the product whose quality can be either low or high. It violates states dominance as the latter option will result in either the same (low) quality as the former option, or in the better (high) quality. Yet, buyers with high b will prefer the former. Our results, however, do not hinge on the violation of state-dominance as state-dominance holds in the expected-utility framework.

Sellers maximize their expected profits, which is

$$U_s = \Pr(\text{sale}) \cdot p,$$

where p is the product price and there is no costs of production (see Gabszewicz and Thisse, 1979; Shaked and Sutton, 1982; and Board, 2009).

This is a one-shot game with three states. During the first stage, both sellers simultaneously send a message m where $m \in \{L, H\}$. Both messages are public and, as mentioned earlier, are free to send regardless of the quality. We interpret message L as disclosing negative information, while message H as not disclosing. In what follows, we will focus on equilibria where high-quality sellers choose to send message H with probability 1. Low-quality sellers, on the other hand, can choose to disclose negative information with a positive probability. In these equilibria, if low-quality sellers choose to disclose negative information, their disclosure is credible in that it becomes common knowledge that the seller's quality is low. In what follows we will refer to sellers who send message L as L-sellers, and sellers who send message H as H-sellers. Given our focus, all L-sellers have the low-quality product; H-sellers can be either high- or low-quality sellers.

At the second stage, given the two announcements sellers determine prices for their products. Let seller i have type t_i. Seller's pricing strategy, $p(m_i, m_j, t_i)$, is a function of his type, message m_i sent by seller i, and message m_j sent by seller j. At the last stage, the degree of buyer's uncertainty aversion, b or γ, is drawn. The buyer has correct beliefs, observes both messages and both prices, and decides from which seller to purchase. Once the buyer purchases the product, the game ends.

A strategy for seller i is a pair $(m_i, p(m_i, m_j, t_i))$, where $m_i \in \{L, H\}$. Buyer's strategy, $s : \{m_i, m_j\} \times \{p_i, p_j\} \to \{i, j\}$, is a binary decision of whether to purchase the product from seller i or j based on sellers' messages and prices.

3.2 Uniform Distribution Example. Link to Hotelling Model

Consider an example of a voluntary negative information disclosure when buyers' attitude towards uncertainty is captured by the perceived-risk framework, and buyers' risk sensitivity, b, is uniformly distributed, $b \sim U[0, B]$.

Using backward induction we start with the last stage, which is the buyer's decision. The buyer with risk-sensitivity b observes the two messages sent by the sellers, as well as the two prices. All high-quality sellers send message H. The buyer believes (correctly in equilibrium) that the low-quality seller discloses negative information with probability λ. Let q_{LH} denote the probability of getting low quality product from the H-seller. Given λ it is equal to:

$$q_{LH} = \frac{(1-\lambda)(1-q)}{(1-\lambda)(1-q)+q}.$$

If both sellers send the same message then the two products are identical from the buyer's point of view and the buyer will purchase the cheapest product.[6] In the asymmetric case when the sellers send two different messages the buyer is indifferent between the two if his risk-sensitivity satisfies:

$$v_L - p_L = q_{LH}v_L + (1-q_{LH})v_H - p_H - b^0 q_{LH}\Big(v_L - q_{LH}v_L - (1-q_{LH})v_H\Big). \tag{3}$$

Here, the term on the left is utility from purchasing product from the L-seller, the term on the right is utility from purchasing from the H-seller. The expected quality of the L-product is v_L; the expected quality of the H product is $q_{LH}v_L + (1-q_{LH})v_H$. The L product has no risk associated with the purchase as its quality is known. If the quality of the H product is high the buyer experiences no loss. If it is low then the size of the loss is $v_L - Ev$. The probability of a loss is q_{LH}. b^0 is the risk-sensitivity of the indifferent buyer. Buyers with $b > b^0$ will purchase from the L-seller and buyers with $b < b^0$ will purchase from the H-seller.

Using the indifference condition above we can relate our model to the Hotelling linear city model. Let $\Delta v = v_H - v_L$. With some algebra we can re-write (3) as

$$\begin{aligned}(1-q_{LH})v_L - p_L - \left[(1-q_{LH})\Delta v - (1-q_{LH})\Delta v\frac{b^0 q_{LH}}{2}\right] &= \\ = (1-q_{LH})v_L - p_H - (1-q_{LH})\Delta v \quad &+ \quad \left[(1-q_{LH})\Delta v - (1-q_{LH})\Delta v\frac{b^0 q_{LH}}{2}\right].\end{aligned}$$

Let $z^0 = (1-q_{LH})\Delta v - (1-q_{LH})\Delta v\frac{b^0 q_{LH}}{2}$. Then the indifference condition above becomes

$$(1-q_{LH})v_L - p_L - z^0 = (1-q_{LH})v_L - p_H - \Big((1-q_{LH})\Delta v - z^0\Big). \tag{4}$$

To see the relation with the Hotelling model consider a linear city model where all buyers have the same value of the product, which is $(1-q_{LH})v_L$. Term $z = (1-q_{LH})\Delta v - (1-q_{LH})\Delta v\frac{bq_{LH}}{2}$ is a geographic location of a given customer with risk-sensitivity b. Given that $b \sim U[0, B]$ we have that $z \sim U[(1-q_{LH})\Delta v(1-\frac{Bq_{LH}}{2}), (1-q_{LH})\Delta v]$. For notational simplicity, denote the lower boundary

[6] We have analyzed what happens if there is preference-based horizontal differentiation between the two sellers. The results are available upon request. If the preferences' strength, t, is large enough there is no equilibrium with information disclosure. The only benefit of information disclosure is to introduce product differentiation; however, for large t this benefit is not strong enough. For small t, it can go either way and low-quality sellers can be either more or less likely to reveal their type.

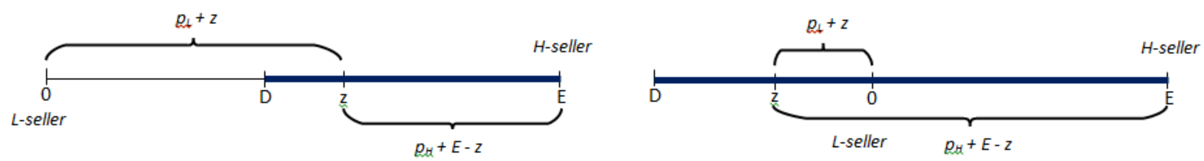

Figure 1: Link to the Hotelling model. Customers are located along interval $[D, E]$ (thick line). L-seller is located at 0, H-seller is located at E. The left picture is the $D > 0$ case, the right picture is the $D < 0$ case. For a customer located at z, the total cost of purchasing from L-seller is $p_L + z$, and from H-seller is $p_H + (E - z)$. When $D > 0$ then L-seller is disadvantaged, as it's located too far from the customers. When $D < 0$ then L-seller is advantaged. Not only there are more customers closer to him, but also when $z < 0$ the customers receive travel subsidy when purchasing from L and travel cost when purchasing from H.

of the support as D and the upper boundary as E so that $z \sim U[D, E]$. D and E determine the boundaries of the city with the left-most buyer located at D and the right-most buyer at E. The L-seller is located at $z_L = 0$, and the H-seller is located at E, $z_H = E$. The travel cost is equal to 1 regardless of the model parameters. The size of the city, $E - D$, depends on B, q and Δv.

The source of product differentiation is the difference in uncertainty regarding the quality. The L-product, the one with negative information disclosed, has lower but certain quality. The H-product, the one without negative information disclosed, has higher but uncertain quality. Buyers vary in terms of their risk-sensitivity so that, *ceteris paribus*, those with lower risk-sensitivity prefer the H-product, while those with higher risk-sensitivity prefer the L-product. Speaking figuratively, for risk-sensitive buyers the known devil (L-product) is better than the unknown angel (the higher but uncertain quality H-product).

The interplay between negative information and product differentiation can be accentuated further if we look at how our setup differs from the Hotelling model (see Figure 1). There are two differences. First, while the H-seller is always at the right end of the city, the L-seller is not necessarily at the left end. When $D > 0$ the L-seller, whose location is 0, is outside of the city. The L-seller is disadvantaged as compared to the H-seller as more buyers are close to the H-seller. When $D < 0$ the L-seller is inside the city. It has an advantage over the H-seller as more buyers are close to the L-seller.

The second difference from the Hotelling model is that when $D < 0$ then for customers located to the left of the L-seller, i.e. those with $z \in [D, 0]$, the further they are from 0, the *lower* their travel cost to the L-seller and the *higher* their travel cost to the H-seller (see (4)). In other words, customers to the left of the L-seller receive travel subsidy when purchasing from the L-seller and bear travel cost when purchasing from the H-seller. The intuition is straightforward. Buyers to the left of $z_L = 0$ are very risk-sensitive. The more risk-sensitive you are, the more valuable is the L-product and the less valuable is the H-product.

When $D < 0$, the L-seller gets a double advantage of having a more profitable location (inside the city) and having customers who have negative travel cost associated with purchasing from L. By definition of D, D is a decreasing function of Bq_{LH} and $D < 0$ whenever $Bq_{LH} > 2$. We will show in Proposition 3 that in equilibrium Bq_{LH} is an increasing function of B. As one would expect, having more risk-sensitive buyers makes disclosure of negative information more profitable.

Next, we solve the second stage of the game and determine equilibrium prices. In the symmetric case when both announcements are the same, the products are not differentiated. The buyer purchases the cheapest product and the Bertrand competition results in both firms charging $p_1 = p_2 = 0$ and earning zero profit. Consider now the asymmetric case, LH. Assuming the interior solution, i.e. $z^0 \in (D, E)$, we have that $p_L = E - \frac{4}{3}D$ and $p_H = E - \frac{2}{3}D$. The location of the indifferent consumer is $z^0 = (1/2)E + (1/3)D$, and the market share of the L-firm is $\frac{1}{2} - \frac{1}{6}\frac{D}{E-D}$.[7] Note, that it is possible that $p_L > p_H$, which happens when $D < 0$.[8]

Now we will, somewhat informally, discuss the conditions that guarantee existence of equilibrium with negative information disclosure. We postpone the formal analysis until the next section. The prices above were derived under the assumption that $z^0 \in (D, E)$, so that the two firms split the market. Condition $z^0 < E$ is always satisfied. Condition $z^0 > D$ is satisfied if and only if $E > (4/3)D$, which is equivalent to $2Bq_{LH} > 1$. As $q_{LH} \in [0, 1-q]$ one can re-write this condition as $2B(1-q) > 1$. If this condition is violated then equilibrium with negative information will not exist. The two firms will not split the customers, as all customers will prefer the H-firm. If $2B(1-q) > 1$ then either B is sufficiently high or q is sufficiently low. The former ensures that there are sufficiently many risk-sensitive customers willing to pay at premium for low but certain quality product. The latter ensures that expected quality of the H-product is not too high which lessens the adverse impact of disclosing the negative information.

Finally, we can solve the first stage of the game. All high-quality sellers send message H. The low-quality seller sends message L with probability λ. In equilibrium, a low-quality seller must be indifferent between sending messages L and H assuming that the other seller sends H if his type is high and sends L with probability λ if his type is low:

$$(1-q)\lambda\pi_{LL} + (1-(1-q)\lambda)\pi_{LH} = (1-q)\lambda\pi_{HL} + (1-(1-q)\lambda)\pi_{HH}.$$

Here π_{ij}, where $i, j \in \{L, H\}$, is a profit of a seller that announces i when the other seller announces j. The LHS is the expected profit from announcing L, and the RHS is the expected profit from announcing H. The probability that the other seller is low-quality and will announce L is $(1-q)\lambda$. With complementary probability the other seller will announce H. Given our assumption that the product cost is the same regardless of the quality, the high-quality seller is also indifferent between

[7] The indifferent consumer is located at $z^0 = (p_H - p_L + E)/2$. The optimization problems are

$$\max_{p_1} \frac{1}{E-D} p_1 \left(\frac{p_H - p_L + E}{2} - D \right),$$

and

$$\max_{p_2} \frac{1}{E-D} p_2 \left(E - \frac{p_H - p_L + E}{2} \right).$$

The solutions to FOCs are $p_1 = (p_2 + E)/2 - A$ and $p_2 = (p_1 + E)/2$. Solving it we get $p_1 = E - \frac{4}{3}D$ and $p_2 = E - \frac{2}{3}D$.

[8] This is due to the fact that, as mentioned earlier, the perceived risk framework violates state dominance. The L-product can be more expensive when there are sufficiently many risk-sensitive buyers.

sending L and H. In particular, sending H with probability 1 is optimal.[9] Firms' profits in the symmetric cases, i.e. after LL or HH announcements, are zero. Given the expressions for p_L, p_H and z that we found earlier, firms' profits in the asymmetric case are

$$\pi_{LH} = \frac{1}{9}(1-q_{LH})\Delta v \frac{(1-2q_{LH}B)^2}{q_{LH}B} \qquad \pi_{HL} = \frac{1}{9}(1-q_{LH})\Delta v \frac{(1+q_{LH}B)^2}{q_{LH}B}. \tag{5}$$

The indifference condition for the low-quality seller becomes:

$$(1-(1-q)\lambda)(1-2q_{LH}B)^2 = (1-q)\lambda(1+q_{LH}B)^2, \tag{6}$$

so that

$$\lambda = \frac{1}{1-q}\frac{(1-2q_{LH}B)^2}{(1-2q_{LH}B)^2+(1+q_{LH}B)^2}, \tag{7}$$

and

$$q = (1-q_{LH})(1-\lambda(1-q)) = (1-q_{LH})\frac{(1+q_{LH}B)^2}{(1-2q_{LH}B)^2+(1+q_{LH}B)^2}. \tag{8}$$

One can use (7) and (8) to solve for λ and q_{LH}.

At the first stage the trade-off faced by the low quality seller is as follows. The asymmetric case is the only case when both sellers make positive profit. When too many low-quality sellers reveal their type, it might be optimal to mimic the high type and announce H. This will maximize the chance of the asymmetric outcome during the second stage. If, on the other hand, too many low-quality sellers choose not to reveal their type then it is better to send message L. In equilibrium low-quality sellers are indifferent between the two messages.

Example 1 *Assume that $B = 56/15$, $q = 2/5$ and $v_H - v_L = 3$. In equilibrium: $\lambda = 3/5$, $q_{LH} = 3/8$, the risk-sensitivity of the indifferent consumer is $b^0 = 32/15$, prices are $p_L = 9/8$ and $p_H = 3/2$.*

If we are to represent the equilibrium above using the linear city analogy from the above, buyers preferences are in interval $[D, E]$ where $D = 9/16$ and $E = 3/2$. Since $D > 0$ the H-firm has advantage in the asymmetric case where the sellers make different announcements (LH). The H-firm charges a higher price and has a higher market share, $b^0/B \approx 57\%$. The low-quality sellers choose to disclose negative information with probability $\lambda = 3/5$. Even though the H-announcement results in a higher profit under the asymmetric case, disclosing negative information is optimal as it allows to avoid zero profits associated with HH-announcements. Value of $\lambda = 3/5$ is when the low-quality sellers are indifferent between the L and H announcements.

Example 2 *Assume that $B = 3$, $q = 1/10$ and $v_H - v_L = 3$. In equilibrium: $\lambda \approx 0.61$, $q_{LH} \approx 0.78$, the risk-sensitivity of the indifferent consumer is $b^0 \approx 1.42$, prices are $p_L \approx 0.81$ and $p_H \approx 0.74$.*

[9] In our alternative interpretation where L means credibly disclosing negative information and H means not, the outcome is the same. High-quality sellers do not have any negative information to disclose and, therefore, have no choice and always send message H.

In this example, the linear city boundaries, $[D, E]$, are $[-0.11, 0.67]$ so that $D < 0$. That gives the L-seller advantage in the asymmetric case. It charges a higher price and its market share is $1 - (b^0/B) \approx 53\%$.

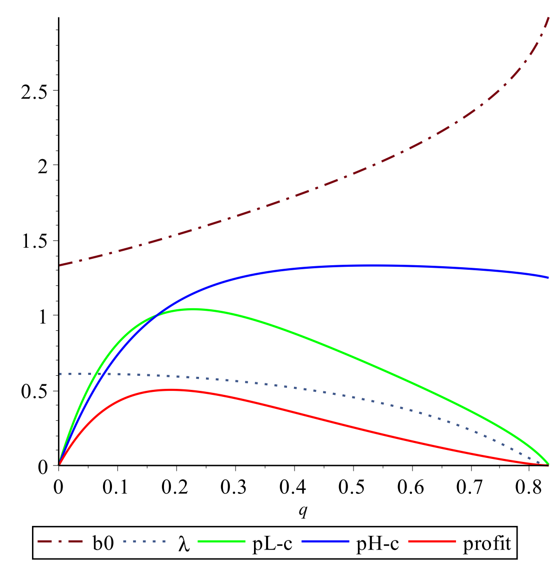

Figure 1: Equilibrium parameters for different values of q. We assume $b \sim U[0, 3], v_H - v_L = 3$.

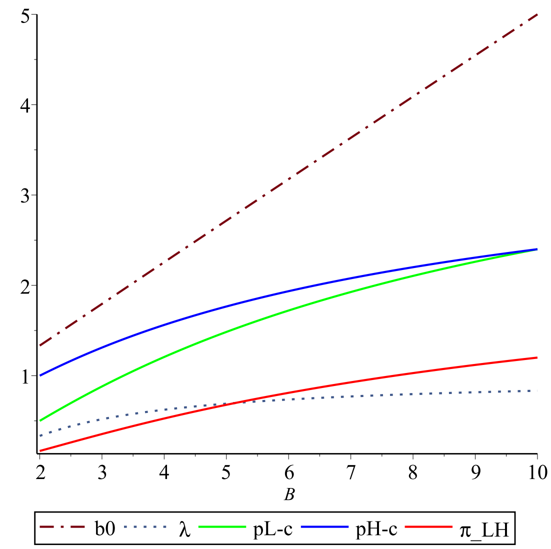

Figure 2: Equilibrium of the model for different values of B. We assume $b \sim U[0, B]$, $v_H - v_L = 3$. The share of high type sellers is $q = 2/5$.

Figures 1 and 2 show how equilibrium changes as we vary q and B. The first figure is plotted given parameter values $v_H - v_L = 3$ and $B = 3$. The second figure is plotted given parameter values $v_H - v_L = 3$ and $q = 2/5$. For Figure 1, q varies between 0 and 5/6, which is the range of q where equilibrium with negative information exists (the exact condition being $2B > 1/(1 - q)$).

One can see from Figure 1 that b^0 is an increasing function of q. Both prices and the profit are non-monotone functions of q. We also see that for low values of q it is possible to have $p_L > p_H$. Notice that since low-quality seller is indifferent between L- and H-announcements and that since costs of low-quality and high-quality sellers are the same, the expected profit is the same for all three groups of sellers. Overall, the industry gains the most from disclosing the negative information for intermediate values of q, which is when disclosure is most informative. For extreme values of q the profit becomes close to zero.

Figure 2 shows how equilibria changes as B goes up. In the case of uniform distribution higher B means that perceived risk is on average viewed more negatively in the population. This has positive effect on the expected profit, as well as prices, as it increases product differentiation, which results in softer competition, higher prices and higher profits.

We conclude this section with comparative statics regarding the three model parameters: Δv, B and q.

Proposition 1 (Comparative static with respect to Δv) *Whether the equilibrium with negative information exists does not depend on Δv. The share of low-quality sellers who disclose* *negative information, λ, does not depend on Δv. Equilibrium prices in the asymmetric case are increasing linear functions of Δv. Equilibrium expected profits of low- and high-quality sellers are increasing linear functions of Δv.*

Proposition 1 shows that larger difference in products' qualities results in larger equilibrium profits. One would not find it surprising that when the quality difference between a high- and low- quality products increases, high-quality sellers benefit more from revealing their type. It turns out, however, that the same is true for low-quality sellers: *a larger quality difference, Δv, makes disclosure of negative information more profitable for low-quality sellers.* Thus, even when high-quality sellers do not or cannot credibly reveal their type, low-quality sellers will have incentive to do so.

Proposition 2 (Comparative static with respect to B) *λ is an increasing functions of B. Prices, p_L and p_H, are increasing function of B. π_{LH} is an increasing function of B; and π_{HL} becomes an increasing function of B once $q_{LH}B > 0.65$. The market share of the H-firm is a declining function of B.*

The intuition is straightforward. Having more risk-sensitive customers in the distribution has two effects: the H- and L-products are more differentiated, and more customers find the L-product attractive. That implies that low-quality sellers will have more incentives to reveal their type. Thus λ goes up, and being an L-seller is beneficial when B is higher. Thus the L-seller's profit and market share in the asymmetric-case go up. The H-seller profit is not monotone, because for low values of B the positive effect of higher prices is outweighed by a negative effect of losing the market share to the L-seller. Once B is large, however, the effect of product differentiation becomes strong enough so that π_{HL} is increasing as well.

Proposition 3 (Comparative static with respect to q) *Probability q_{LH} is a decreasing function of q. The product differentiation as measured by the city length, $E - D$ is the longest when $q_{LH} = 1/2$ and is a single-peaked function of q. Firms' profits and prices are non-monotone functions.*

The intuition is as follows. In our model the product differentiation is caused by the difference in risk associated with purchasing L- and H-products. The L-product has no risk, and when q is either high or low then the risk associated with H-product is also low. For very low or very high values of q, the negative information does not generate sufficient product differentiation, as the risk associated with H-product is too low. Either it's very likely to be low-quality product (when q is low) or high-quality product (when q is high). In terms of posterior probability, q_{LH}, the risk is the highest when $q_{LH} = 1/2$ as there is 50-50 chance of getting low quality product from the H-seller, which is when the city's length is the highest. As product differentiation goes down, so do prices and profits.

4 The general case.

4.1 Equilibrium System

Buyer's stage. We solve the model using backward induction. We start from the last stage, which is the buyer's choice of the seller. We assume that a low-quality seller sends message L with probability $\lambda \in [0,1]$, a high-quality seller sends message H, and sellers set prices according to their pricing strategies: $p_{t_i}(m_i, m_j)$. Buyer's choice will depend on buyer's beliefs which are functions of messages and prices.

Let $\mu(p)$ denote the buyer's belief regarding the probability of a seller who charges price p to be a low-type. For any (m_i, m_j) there can be at most three on-equilibrium prices. Price set by the high-type, p_H, price set by low-type who announced H, p_{LH}, and price set by low-type who announced L, p_L. We will be interested in equilibria where $p_L \neq p_{LH} = p_H$. This is because if the low-type H-announcer charges price different from p_H, then $\mu(p_{LH}) = 1$ and $\mu(p_H) = 0$. The low-quality sellers' type is revealed and this case is equivalent to the case of $\lambda = 1$.

When $p_L \neq p_{LH} = p_H$, prices are not informative about H-seller's quality. Buyers' on-equilibrium beliefs, therefore, are $\mu(p_L) = 1$ and $\mu(p_H) = \lambda$. There are three cases with the most interesting case being an asymmetric case, where sellers make different announcements (LH). We will handle the (LH) case separately for the perceived risk, $(LH - PR)$, and expected utility, $(LH - EU)$, frameworks.

[LL] Both sellers disclose negative information by sending message L. Then both sellers have the same quality L, and there is no perceived risk associated with purchasing from either seller. The buyer will purchase from a seller with the lowest price. The buyer's utility is

$$v_L - \min\{p_1, p_2\}$$

in the perceived-risk framework, and

$$u(v_L - \min\{p_1, p_2\})$$

in the expected utility framework.

[HH] Both sellers choose not to disclose negative information and send message H. Given the strategy of L-sellers, the probability of acquiring the low quality product from the H-announcer, q_{LH}, is

$$q_{LH} = \frac{(1-\lambda)(1-q)}{(1-\lambda)(1-q)+q}.$$

The expected quality from either seller is $E[v|H] = q_{LH}v_L + (1 - q_{LH})v_H$, and the buyer will purchase a product from the seller with the lowest price. Under the perceived-risk framework, the buyer's utility is

$$E[v|H] - \min\{p_1, p_2\} + bq_{LH}(v_L - E[v|H]),$$

and under the expected-utility framework, it is

$$E_v[u(v - \min\{p_1, p_2\})|H].$$

[LH-PR] Sellers make different announcements. Let p_L denote the price of the seller who announced L, and p_H denote the price of the seller who announced H. Buyer's utility of purchasing from the L-announcer is $v_L - p_L$. There is no risk, because the quality of L-announcer is known to be low. Purchasing from the H-announcer gives utility of

$$E[v|H] - p_H + b \cdot E(v - E[v|H]|v \leq E[v|H]),$$

which is equal to

$$q_{LH}v_L + (1 - q_{LH})v_H - p_H + b \cdot q_{LH}(v_L - q_{LH}v_L - (1 - q_{LH})v_H)$$

A buyer will be indifferent between two sellers if his b^0 is such that

$$v_L - p_L = q_{LH}v_L + (1 - q_{LH})v_H - p_H + b \cdot q_{LH}(v_L - q_{LH}v_L - (1 - q_{LH})v_H), \tag{9}$$

and, therefore,

$$b^0(p_L, p_H) = \frac{1}{q_{LH}} - \frac{p_H - p_L}{v_H - v_L}\frac{1}{q_{LH}}\frac{1}{1 - q_{LH}}. \tag{10}$$

All buyers with $b < b^0$ will purchase from the H-announcer and all buyers with $b > b^0$ will purchase from the L announcer.

[LH-EU] Buyer's expected utility of purchasing from the L-announcer is $u(v_L - p_L)$. There is no risk, because the quality of L-announcer is known to be low. Purchasing from the H-announcer gives expected utility of

$$q_{LH}u(v_L - p_H) + (1 - q_{LH})u(v_H - p_H).$$

A buyer will be indifferent between two sellers if his absolute risk-aversion, γ^0, is such that

$$u(v_L - p_L) = q_{LH}u(v_L - p_H) + (1 - q_{LH})u(v_H - p_H). \tag{11}$$

Proposition 6 will show that all buyers with $\gamma < \gamma^0$ will purchase from the H-seller and all buyers with $\gamma > \gamma^0$ will purchase from the L-seller.

The buyer's stage highlights costs and benefits of disclosing the negative information. On the one hand, disclosing the low quality means that buyers have a lower valuation of the seller's product. On the other hand, it gives a chance for low-quality sellers to differentiate their product and soften the competition. When the buyer has identical beliefs about quality of both sellers, their products are homogeneous and the buyer then purchases the product with the lowest price. The ensuing pricing competition results in the Bertrand outcome where both sellers receive zero profit and the equilibrium prices are c. In the asymmetric case, the sellers offer different products from the buyers' point of view. The L-product has lower but certain quality; the H-product has higher but uncertain quality. Buyers' with low b (low γ) value quality and are more likely to go for the H-announcer's product. Buyers with high b (higher γ) dislike risk and would rather get product with a lower but certain quality. That introduces differentiation in the sellers' products and softens the price competition between the two sellers.[10]

Second stage. Optimal prices Cases (LL) and (HH) are similar. The buyer views products offered by the two sellers as identical and will purchase from the seller who offers the lowest price. Both sellers will set the price equal to marginal cost and will earn zero profit. The buyer buys from either seller with probability 1/2.

Next we look at the (LH-PR)-case. Case (LH-EU) is identical with the only exception that one should use γ instead of b. As established above, buyers with $b > b^0(p_L, p_H)$ will purchase the product from the L-seller and buyers with $b < b^0(p_L, p_H)$ will purchase the product from the H-seller. The sale probability after announcement L is $(1 - \Phi(b^0(p_L, p_H))$, and after announcement H is $\Phi(b^0(p_L, p_H))$. The optimization problem of the L-announcer is

$$\max_{p_L}(1 - \Phi(b^0(p_L, p_H)) \cdot p_L,$$

and the optimization problem of the H-announcer is

$$\max_{p_H} \Phi(b^0(p_L, p_H)) \cdot p_H.$$

The corresponding first-order conditions are

$$-\phi(b^0)\frac{\partial b^0(p_L, p_H)}{\partial p_L}p_L + (1 - \Phi(b^0)) = 0. \tag{12}$$

and

$$\phi(b^0)\frac{\partial b^0(p_L, p_H)}{\partial p_H}p_H + \Phi(b^0) = 0. \tag{13}$$

First stage. Disclosing the negative information. We are focusing on equilibria where high-quality sellers announce H with probability 1. The low quality seller chooses to disclose negative information with probability $\lambda \in [0, 1]$. There are three cases: $\lambda = 1$, $\lambda = 0$ and $0 < \lambda < 1$. If $\lambda = 1$ then in the (LH) case the perceived risk is zero in both cases. The (LH)-case results in equilibrium with $p_L = 0$ and $p_H = v_H - v_L$. The low-quality sellers then would have incentives to

[10]The logic here can be related to Board (2009). The case (HH) correspond to the "*Neither firm discloses*" case in Board (2009) where both firms are identical and, as the result, earn zero profits. The (LH) case is related, though different, to equilibrium with partial disclosure which result in product differentiation. The primary difference is that in Board's model both firms can disclose, and in equilibrium either both firms or only a higher-quality firm will disclose. In our model high quality sellers cannot disclose. However, the motivation for low quality sellers to disclose is similar: to introduce product differentiation.

deviate by announcing H and enjoying a higher price p_H. If in equlibrium $\lambda = 0$, then the negative information is not disclosed, and both sellers announce H regardless of their type.

Finally, consider the case $0 < \lambda < 1$. In order for the low-quality seller to be willing to randomize, his expected profit from announcement L should be equal to his profit from announcement H:

$$\lambda(1-q)\pi_{LL} + (1-\lambda(1-q))\pi_{LH} = \lambda(1-q)\pi_{HL} + (1-\lambda(1-q))\pi_{HH}, \tag{14}$$

where π_{LH} is the profit from announcing L when the other seller announces H. π_{HL}, π_{LL}, and π_{HH} are expected profits when sellers announce (HL), (LL), and (HH). The probability that the other seller will announce L, which we will denote as q_L, is $\lambda(1-q)$. It is equal to the probability that the other seller is the low-quality seller, $1-q$, times the probability with which the low-quality seller chooses to disclose negative information, λ. All high-quality sellers and $(1-\lambda)$ of low-quality sellers will announce H. That for high-quality sellers it is optimal to announce H follows from the fact that since both high- and low-quality products have the same cost expected profit associated with an announcement, whether L or H, is the same for both high- and low-quality sellers. Then high-quality sellers are also indifferent between L and H. and, therefore, announcing H is optimal.[11]

We know that $\pi_{LL} = \pi_{HH} = 0$ and, therefore, the indifference condition (14) can be re-written as $(1-q_L)\pi_{LH} = q_L\pi_{HL}$. For the perceived-risk framework it can be re-written as

$$(1-q_L)(1-\Phi(b^0))p_L = q_L\Phi(b^0)p_H, \tag{15}$$

and for the expected-utility framework as

$$(1-q_L)(1-\Phi(\gamma^0))p_L = q_L\Phi(\gamma^0)p_H. \tag{16}$$

4.2 Disclosure of Negative Information. Perceived Risk Framework

Combining all the steps above we conclude that in the perceived risk framework, equilibrium parameters (λ, b^0, p_H, p_L) must satisfy equations (10), (12),(13) and (16). The equilibrium system thus can be written as

$$\begin{cases} b^0 = \dfrac{1}{q_{LH}} - \dfrac{p_H - p_L}{v_H - v_L}\dfrac{1}{q_{LH}}\dfrac{1}{1-q_{LH}} \\ -\phi(b^0)\dfrac{\partial b^0(p_L, p_H)}{\partial p_L}p_L + (1-\Phi(b^0)) = 0 \\ -\phi(b^0)\dfrac{\partial b^0(p_L, p_H)}{\partial p_L}p_H + \Phi(b^0) = 0 \\ (1-q_L)(1-\Phi(b^0))p_L = q_L\Phi(b^0)p_H, \end{cases} \tag{17}$$

where recall $q_L = \lambda(1-q)$ is a probability of an L announcement by a given seller, and q_{LH} is probability of a seller being low-quality conditional on announcement H. For the third equation, we re-wrote (13) taking into account that $\partial b^0(p_L, p_H)/\partial p_H = -\partial b^0(p_L, p_H)/\partial p_L$.

[11] As mentioned in the case of the uniform distribution, for our alternative interpretation where L means a credible disclosure of negative information, high-quality sellers have no choice but to announce H as they have no negative information to disclose. Thus, it is still the case that they announce H with probability 1.

The next lemma will allow us to temporarily ignore the fourth equation of (17) and treat q_{LH} as given. We will refer to a system that consists of the first three equations of (17) as *reduced system.* Once we solve the reduced system for (b^0, p_L, p_H), we can recover values of q and λ from the fourth equation.

Lemma 1 *If for a given $q_{LH} \in [0,1]$ the solution $(\bar{b}^0, \bar{p}_L, \bar{p}_H)$ to the reduced system exists, then there exists $q \in [0,1]$ and $\lambda \in [0,1]$ such that $(\bar{b}^0, \bar{p}_L, \bar{p}_H, \lambda)$ is a solution to the equilibrium system (17).*

Solving the reduced system is fairly straightforward. Subtracting the third equation from the second equation we get

$$\frac{\Phi(b^0)}{\phi(b^0)} - \frac{1-\Phi(b^0)}{\phi(b^0)} = (p_H - p_L)\frac{\partial b^0(p_L, p_H)}{\partial p_L}.$$

Denote the LHS of the equation above as $A(b^0)$. We will interpret this term later, once we derive the equilibrium. From the indifference condition we get that

$$\frac{\partial b^0(p_L, p_H)}{\partial p_L} = \frac{1}{v_H - v_L}\frac{1}{q_{LH}(1-q_{LH})}.$$

Thus, we have

$$A(b^0) = \frac{p_H - p_L}{v_H - v_L}\frac{1}{q_{LH}(1-q_{LH})}.$$

Taking into account that

$$b^0 = \frac{1}{q_{LH}} - \frac{p_H - p_L}{v_H - v_L}\frac{1}{q_{LH}}\frac{1}{1-q_{LH}},$$

we get that the risk-sensitivity of the indifferent consumer should satisfy

$$b^0 + A(b^0) = \frac{1}{q_{LH}}. \tag{18}$$

Therefore, the equilibrium with negative information exists if and only if (18) has solution. Given that $q_{LH} \in [0, 1-q]$, we conclude that the equilibrium with negative information exists if and only if

$$\max_{b\in[0,B]} \Big\{b + A(b)\Big\} > \frac{1}{1-q}. \tag{19}$$

Proposition 4 summarizes our findings:

Proposition 4 *Equilibrium with negative information exists if and only if* $\max_b \Big\{b+A(b)\Big\} > \dfrac{1}{1-q}$.

We can now interpret Proposition 4. In (19) the term on the left is determined by the distribution of risk-sensitivity among buyers, and the term on the right is determined by the distribution of quality among sellers. First, for given $\Phi(b)$ whether the equilibrium exists or not depends on q. Let q^0 be such that

$$\max_b \Big\{b + A(b)\Big\} = \frac{1}{1-q^0}.$$

Then, it follows from (19) that for the equilibrium with negative information to exist it has to be the case that $q < q^0$. When the share of high-quality sellers is too high then one can't have negative information disclosure. High q means that there is a higher proportion of high-quality sellers and, therefore, the risk of getting a low-quality product from H-announcers is low.

Next we fix q. In order for the equilibrium with negative information to exist the LHS of (19), $\max_{b\in[0,B]}\Big\{b + A(b)\Big\}$, should be large enough to be above $1/(1-q)$. There are two sufficient conditions either of which will guarantee the existence of the equilibrium: either the support of $\Phi(b)$ is large enough, or function $\Phi(b)$ is such that the maximum of $A(b)$ is large enough. The intuition behind the first condition is straightforward. Disclosure of negative information attracts risk-sensitive customers, as it removes uncertainty regarding the product quality. Customers have to be sufficiently risk-sensitive in order to opt for certain lower quality over uncertain higher quality. Thus, as long as support of b is sufficiently large, e.g. $B \geq 1/(1-q)$, the equilibrium with negative information exists.

The intuition behind the second condition shows why high value of B is not necessary for the equilibrium with negative information to exist. Term $A(b)$ is directly related to marginal revenues of both sellers. For the L-seller the marginal revenue is

$$MR_L = p_L - \frac{1-\Phi(b^0)}{\phi(b^0)\cdot(\partial b^0/\partial p_L)},$$

and for the H seller it is

$$MR_H = p_H - \frac{\Phi(b^0)}{\phi(b^0)\cdot(-\partial b^0/\partial p_H)}.$$

The first terms are marginal gains from serving an extra customer. The second terms are marginal losses from charging a lower price to existing customers. Term $A(b)$ thus is proportional to the difference between revenue lost the H-seller versus the L-seller from a marginal expansion in output. High $A(b)$ means that comparatively to the L-seller, the H-seller's expansion is costlier and will result in a higher loss. It means that the H-seller has less incentives to outprice the L-seller from the market, so that the L-seller ends up with a positive market share and a positive profit.

There are two notable cases when $A(b)$ becomes sufficiently large. First, it occurs when the risk-sensitivity distribution is bi-modal. More precisely, when the risk-sensitivity of most buyers belongs to two disjoint intervals $[0, b_1]$ and $[b_2, B]$ so that $\phi(b)$ is low when $b \in [b_1, b_2]$ and $\Phi(b_1) > 1/2$. Given that $A(b) = (2\Phi(b)-1)/\phi(b)$ and that $2\Phi(b)-1$ is bounded, we then have that $A(b)$ is sufficiently large when $b \in [b_1, b_2]$. The split of buyers' preferences into two intervals generates a natural market split. For the H-seller the price reduction required to attract buyers with $b > b_2$ can result in too much lost revenue from the customers on $[0, b_1]$.

Second, consider two distribution $\Phi(b)$ and $\Psi(b)$ such that $\Psi(b)$ dominates $\Phi(b)$ in terms of likelihood ratio (see Krishna, 2009). When this is the case then

$$A_\Phi(b) = \frac{2\Phi(b)-1}{\phi(b)} > \frac{2\Psi(b)-1}{\psi(b)} = A_\Psi(b).$$

Thus distributions that are LR-dominated result in higher values of $A(b)$ and, therefore, equilibrium with negative information is more likely to exist. Intuitively, if $\Phi(b)$ is LR-dominated by $\Psi(b)$ then

$\psi(b)/\phi(b)$ is a strictly increasing function of b. It means that $\Phi(b)$ puts more weight on customers with low risk-sensitivity, while $\Psi(b)$ puts more weight on customers with high risk-sensitivity. From H-seller point of view, having more customers with low risk-sensitivity makes competing for customers with high risk-sensitivity less attractive, thereby creating a market segment for the L-sellers. For example, if $\Psi(b) = \Phi(b)^a$, where $a > 1$, then $\Psi(b)$ LR-dominates $\Phi(b)$. When $a > 1$ the transformation $\Phi(b)^a$ puts more weight on customers with high risk-sensitivity. Having more customers with higher risk-sensitivity implies that it is more profitable for the H-seller to compete for them thereby making equilibrium with negative information less likely to exist under $\Psi(b)$.

Example 3 *When $b \sim U[0, B]$ term $A(b)$ is $2b - B$. Condition (19) becomes*

$$\max_{b\in[0,B]} \Big\{ b + (2b - B) \Big\} = 2B > \frac{1}{1-q}. \tag{20}$$

When risk-sensitivity is uniformly distributed, the equilibrium with negative information exists when either B is sufficiently high, or q is sufficiently low so that (20) is satisfied.

4.3 Disclosure of Negative Information. Expected Utility Framework

In this section we model buyers' risk attitude using the expected utility framework. We assume that buyers have a CARA utility, $u(\cdot)$, with an absolute risk-aversion coefficient $\gamma > 0$. Different buyers have different γ and, with a slight abuse of notations, we denote the cdf of γ as $\Phi(\gamma)$.

In the case of risk-averse buyers, the only thing that changes in equilibrium system (17) is the first equation, which is the buyer's indifference condition. The other three equations — which are profit-maximization by both firms, and indifference condition for the low-quality seller — remain unchanged.

A risk-averse buyer with the CARA utility function $u(x) = 1 - e^{-\gamma x}$ is indifferent between L and H products if

$$u(v_L - p_L) = q_{LH} u(v_L - p_H) + (1 - q_{LH}) u(v_H - p_H),$$

or equivalently

$$e^{-\gamma^0(p_H - p_L)} - q_{LH} - (1 - q_{LH}) e^{-\gamma^0(v_H - v_L)} = 0. \tag{21}$$

where γ^0 is the risk-aversion degree of an indifferent buyer.

Thus an equilibrium with disclosure of negative information is determined by

$$\begin{cases} e^{-\gamma^0(p_H - p_L)} = q_{LH} + (1 - q_{LH}) e^{-\gamma^0(v_H - v_L)}. \\ -\phi(\gamma^0) \dfrac{\partial \gamma^0(p_L, p_H)}{\partial p_L} p_L + (1 - \Phi(\gamma^0)) = 0 \\ \phi(\gamma^0) \dfrac{\partial \gamma^0(p_L, p_H)}{\partial p_H} p_H + \Phi(\gamma^0) = 0 \\ (1 - q_L)(1 - \Phi(\gamma^0)) p_L = q_L \Phi(\gamma^0) p_H. \end{cases} \tag{22}$$

From (21) follows that

$$\frac{\partial \gamma^0}{\partial p_L} = -\frac{\partial \gamma^0}{\partial p_H} = -\frac{\gamma^0 e^{-\gamma^0(p_H - p_L)}}{-(p_H - p_L) e^{-\gamma^0(p_H - p_L)} + (1 - q_{LH})(v_H - v_L) e^{-\gamma^0(v_H - v_L)}},$$

so that the equilibrium system becomes

$$\begin{cases} e^{-\gamma^0(p_H-p_L)} = q_{LH} + (1-q_{LH})e^{-\gamma^0(v_H-v_L)}. \\ -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L,p_H)}{\partial p_L}p_L + (1-\Phi(\gamma^0)) = 0 \\ -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L,p_H)}{\partial p_L}p_H + \Phi(\gamma^0) = 0 \\ (1-q_L)(1-\Phi(\gamma^0))p_L = q_L\Phi(\gamma^0)p_H. \end{cases} \tag{23}$$

Recall, that when we used perceived risk to model customer's attitude toward risk it was possible to have $p_H < p_L$. As we explained earlier, that was because perceived risk violates states dominance and low-quality product can be more valuable for risk-sensitive customers. That cannot happen within the expected utility framework which respects state-dominance. As Proposition 5 shows, in equilibrium the product with higher expected quality has higher price.

Proposition 5 *In any equilibrium with a disclosure of negative information $p_H > p_L$.*

The next proposition shows that similarly to the perceived-risk case all buyers with low risk-aversion will prefer a product with higher quality. All buyers with higher risk-aversion will prefer a product with a lower but certain quality.

Proposition 6 *The indifference condition*

$$e^{-\gamma^0(p_H-p_L)} = q_{LH} + (1-q_{LH})e^{-\gamma^0(v_H-v_L)},$$

has at most one solution $\gamma^0 > 0$. When the solution exists, all types with $\gamma > \gamma^0$ prefer an L-product while all types with $\gamma < \gamma^0$ prefer H-product.

Whether the equilibrium with the disclosure of negative information exists or not depends on the distribution of γ. The next Proposition provides conditions on distribution $\Phi(\gamma)$ that either guarantee that such an equilibrium exist or guarantee that it does not exist.

Proposition 7 *Equilibrium with negative information does not exist if*

i) $\Phi(\gamma)$ is a uniform distribution; or

ii) $\Phi(\gamma)$ is a convex function.

The equilibrium with negative information exists if

iii) $\Phi(\gamma)$ has infinite support; alternatively

iv) for any concave $\Phi(\gamma)$ with finite support there exists $\alpha^0 > 0$ such that for any $\alpha \in (0, \alpha^0)$, if risk-aversion is distributed with cdf $\Phi(\alpha\gamma)$ then an equilibrium with negative information exists.

The intuition is as follows. In any equilibrium with negative information H-sellers serve customers with low risk-aversion, and L-sellers serve customers with high risk-aversion. Given that the H-sellers provide a superior product with higher expected quality, they can outprice the L-sellers from the market, if they want to. This is because, in the expected utility framework, other things being equal, H-product is preferred by all buyers. Whether it is optimal to outprice L-sellers depends on how many buyers have high risk-aversion. If their share is low, then H-sellers might choose not to serve those customers and charge a higher price instead. If their share is high, however, the H-sellers will not be willing to give up on those customers.[12] Thus, for any $\Phi(\gamma)$ with infinite support the equilibrium with negative information exists. There will always exist $\tilde{\gamma}$ high enough so that the share of buyers with $\gamma > \tilde{\gamma}$ is too low, making H-sellers less interested in competing for those buyers. This is not the case for distribution with finite support, no matter how large the support is. Consider, for example, $\Phi(\gamma) = (\gamma/\Gamma)^n$ which has support $[0, \Gamma]$. For any Γ, when $n > 1$ there are sufficiently many risk-averse buyers (those at the right side of the support) so that the equilibrium with negative information would not exist.

Proposition 7 captures the notion of having "*sufficiently many risk-averse buyers*" using convexity of $\Phi(\gamma)$. Under uniform or convex distributions, there are sufficiently many buyers with high degree of risk-aversion, making equilibrium with negative information impossible. As for concave distributions, concavity alone is not enough to guarantee existence. One also needs to have sufficiently risk-averse buyers in the support. Otherwise, when the support's upper-bound is too low there might not be enough risk-averse buyers to make the disclosure worthwhile. One way to achieve it is by stretching $\Phi(\gamma)$ to a larger support, which is Proposition 7 does. Notably, one does not need unrealistic levels of risk-aversion for the equilibrium existence.

Example 4 *Let the degree of risk-aversion* γ *be distributed with* $\Phi(\gamma) = \sqrt{\gamma}$ *on* $[0, 1]$*. The highest degree of risk-aversion among buyers, therefore, is 1. Let* $v_H - v_L = 3$ *and* $q \approx 0.484$.[13] *One can verify that the following is equilibrium:* $\gamma^0 \approx 0.713$, $p_L \approx 0.184$, $p_H \approx 1$, $\lambda \approx 0.063$ *and* $q_{LH} = 1/2$.

In this example, the probability of buying from a low-quality seller is slightly above 1/2, $1 - q \approx 0.52$*. The low-quality seller reveals the negative information with probability 6.3%. If one seller announces L and another announces H, then both prices are above the marginal cost and both sellers making positive profit. The indifferent buyer is located at* $\gamma^0 \approx 0.713$ *so that the L-announcer has* $1 - \sqrt{\gamma^0} \approx 15.5\%$ *share of the market and the H-announcer has 84.4% of the market. In the case of asymmetric announcements, sellers profits are* $\pi_H \approx 0.84$ *and* $\pi_L \approx 0.03$.

5 Conclusion

Although conventional wisdom holds that negative information hurts sellers, many sellers still voluntarily share negative information and claim low quality in the markets where information

[12] The logic is similar to that of the perceived risk case with one caveat. In the expected utility case all buyers, other things being equal, prefer the H-product. In the perceived-risk case the H-product is not uniformly superior. Only buyers with low risk-sensitivity prefer the H-product. However, since $b \sim [0, B]$, i.e. there are always buyers with sufficiently low risk-sensitivity, the H can always guarantee to himself a positive market share by matching the price of the L-seller. The only question is whether the H-seller wants to outprice the L-seller entirely. And, as we discussed earlier, it, in part, depends on the share of risk-sensitive buyers.

[13] Numerically, it is simpler to start with q_{LH} instead of q. The equilibrium in this example was calculated based on $q_{LH} = 1/2$. Value of q and λ were then calculated from (γ^0, p_L, p_H) and q_{LH} using the fourth equation of (23).

asymmetry is high. This is somewhat counterintutive as sellers may get damaged by such a disclosure especially when customers cannot easily evaluate true quality in these markets. This paper has analyzed how honesty can be beneficial for low-quality sellers. We have shown that sharing negative information can reduce the expected disutility of risk-sensitive (risk-averse) buyers and increase sales of low-quality products. In equilibrium the negative effect of lowering buyer's valuation is outweighed by the positive effect of reducing perceived risk and softening the competition. Moreover, announcing negative information has been found to increase profits for both low-quality *and* high-quality sellers as all benefit from weaker competition.

To the best of our knowledge, this is the first study that has provided a theoretical framework on the economic incentives for low-quality sellers to reveal its type. This is a novel aspect of our model as a more common source of information disclosure comes from incentives of high-quality sellers. Furthermore, it follows from our model that even if high-quality sellers cannot credibly reveal their high-quality, e.g. when the certification is infeasible or too expensive, the information nonetheless can be disclosed in equilibrium and it is low-quality sellers who disclose it.

We believe that the findings of this paper can be universally applied to numerous real market cases, since our assumption of costless cheap talk is actually the case in many markets where information asymmetry is high. Furthermore, the findings of this paper are expected to provide valuable insights to both the field and the academia on the behaviors of sellers in the markets under information asymmetry. For marketing managers, this paper can provide helpful strategic suggestions on how to deal with the information about the weaknesses of their products. This is an important issue for most marketers, as no product or service is perfect in customers' eyes, and sellers always have to deal with certain negative information. For policy makers, this paper suggests a new perspective on how to effectively deal with frauds in markets where information asymmetry is high. According to our results, the level of market fraud might be affected by several factors, such as the ratio of high-quality products and the level of risk-sensitivity of customers, and thus policy makers might have to measure and monitor those factors in order to cope with potential frauds. For academic researchers, this paper provides a theoretical explanation on the important market phenomena that have been somewhat neglected by the literature. More specifically, this paper suggests that we observe different levels of frauds across markets in different industries or different nations due to varying levels of the factors presented in this paper.

On a final note, we hope that this study provides motivations for further empirical and theoretical research regarding the effect of sharing negative information under various different settings, which can lead to a better understanding of markets under information asymmetry. In particular, empirical research measuring the effect of the factors suggested in our model and analyzing how they affect the inter-industry or inter-country differences in terms of the level of fraud can be one of the important directions for future research.

6 Appendix: Proofs

Proof of Proposition 1: From (7) and (8) follows that Δv does not affect λ and q_{LH}. Since q_{LH} does not depend on Δv we can conclude that D and E are linear functions of Δv. The rest is straightforward. Equilibrium with negative information exists if $E > (4/3)D$ and Δv cancels. Asymmetric-case prices, p_L and p_H, are linear functions of D and E and, therefore, are linear functions of Δv. That asymmetric-case profits are linear functions of Δv follows from (5). Finally, given that λ does not depend on Δv, the expected profit of the low-quality seller, regardless of the message, is $(1-\lambda(1-q))\pi_{LH}$ and is linear function of Δv. The expected profit of the high-quality seller is $\lambda(1-q)\pi_{HL}$ and is also a linear function of Δv. ■

Proof of Proposition 2: When $q_{LH}B > 1/2$, i.e. when the equilibrium exists, it is straightforward to verify that the RHS of (8) is a decreasing function of B and of q_{LH}. Note that (8) determines an implicit relationship between q_{LH} and B. Since the RHS is a decreasing function of both B and q_{LH} it implies that q_{LH} is a decreasing function of B. It is also immediate to show that $q_{LH}B$ is an increasing function of B. This is because, when B goes up q_{LH} must go down $1-q_{LH}$ goes up. In order to maintain (8) it means that the faction should go down, which therefore means that $q_{LH}B$ went up. Given that $q_{LH}B$ is an increasing function of B, we can conclude that λ is also an increasing function of B.

Next we look at prices and profits. If we write equilibrium prices p_L and p_H in terms of model primitives instead of D and E:

$$p_L = \frac{1}{3}(1-q_{LH})\Delta v(2q_{LH}B-1) \qquad p_H = \frac{1}{3}(1-q_{LH})\Delta v(q_{LH}B+1).$$

we see that for both L- and H-announcers prices are increasing functions of $q_{LH}B$ and, therefore, of B. That π_{LH} is an increasing function of B follows from the fact that so is $(1-q_{LH})$ is an increasing function of B and $\frac{(1-2q_{LH}B)^2}{qLHB}$ is an increasing function of $q_{LH}B$ (when $q_{LH}B > 1/2$) and, therefore, an increasing function of B. As for π_{HL}, we will use (8) to express q_{LH} in terms of $q_{LH}B$:

$$1-q_{LH} = q\frac{(1-2q_{LH}B)^2+(1+q_{LH}B)^2}{(1+q_{LH}B)^2}.$$

Then

$$\pi_{HL} = \frac{1}{9}q\Delta v\frac{(1-2q_{LH}B)^2+(1+q_{LH}B)^2}{q_{LH}B}.$$

It is a non-monotone function of $q_{LH}B$. It decreases until $q_{LH}B \approx 0.65$ and increases afterwards. In other words, for small values of B the H-profit is a decreasing function of B, but then it becomes an increasing function as well.

Finally, the market share of the H-announcer, $(E-z^0)/(E-D)$, is a decreasing function of B. Indeed, it is straightforward to verify that

$$\frac{E-z^0}{E-D} = \frac{1}{3}\frac{1+q_{LH}B}{q_{LH}B}.$$

It is equal to 1 when $q_{LH}B = 1/2$ and declines afterwards. ■

Proof of Proposition 3: As before we use (8) to establish that q_{LH} is a decreasing function of q. The length of the city, $E - D$, is equal to $\Delta v(1 - q_{LH})\frac{q_{LH}B}{2}$ and is the longest when $q_{LH} = 1/2$. Since q_{LH} is a monotone function of q so is $E - D$.

For a given B the equilibrium with negative information exists for $q \in [0, 1 - 1/(2B)]$. As q varies from 0 to $1 - \dfrac{1}{2B}$, which is the upper bound for the equilibrium with negative information to exist, q_{LH} declines from 1 to $1/(2B)$. Given the range of q_{LH} one can immediately see that p_L and π_{LH} are non-monotone functions of q that reach 0 at the two borders of the q-range. It's not the case for p_H and π_{HL}. Both are equal to 0 when $q = 0$ (or $q_{LH} = 1$), but both are positive when $q = 1/(2B)$. Whether they are monotone functions or q or not depends on the value of B. When $B \le 1$ it is a strictly increasing function of q. When $B > 1$ is it a non-monotone single-peaked function of q. ■

Proof of Lemma 1: Plug values $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H)$ into the the last equation of (17) to recover q_L. Notice that for any $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H)$ one can find q_L such that it is between 0 and 1 and the last equation is satisfied. From q_L and q_{LH} one can then uniquely recover q and λ: $q = (1 - q_{LH})(1 - q_L)$ and $\lambda = q_L/(1 - q)$. The only thing one has to check is that $\lambda, q \in [0, 1]$ and that $q_{LH} \le 1 - q$. The last inequality states that share of low-quality sellers who announce high quality, q_{LH}, cannot be higher than the total share of low quality sellers, $1 - q$. This is straightforward. That $q_{LH} \le 1 - q$ is trivial:

$$q_{LH} \le 1 - q = 1 - (1 - q_{LH})(1 - q_L) = q_{LH} + q_L - q_{LH} \cdot q_L.$$

Given that $q = (1 - q_{LH})(1 - q_L)$ is it between 0 and 1. Finally, $\lambda < 1$ is equivalent to $q_L < 1 - q$, which is also true.

$$q_L \le 1 - q = 1 - (1 - q_{LH})(1 - q_L) = q_{LH} + q_L - q_{LH} \cdot q_L.$$

This completes the proof. ■

Proof of Proposition 5: Consider the indifference condition

$$e^{-\gamma^0(p_H - p_L)} = q_{LH} + (1 - q_{LH})e^{-\gamma^0(v_H - v_L)},$$

and take natural logarithm of both sides:

$$-\gamma^0(p_H - p_L) = \ln(q_{LH} + (1 - q_{LH})e^{-\gamma^0(v_H - v_L)}).$$

The expression inside the logarithm is less than one. Indeed, it is a weighted average of 1 and $e^{-\gamma^0(v_H - v_L)}$. Term $e^{-\gamma^0(v_H - v_L)} < 1$ because γ^0 and $v_H - v_L$ are positive. Thus $\ln(q_{LH} + (1 - q_{LH})e^{-\gamma^0(v_H - v_L)}) < 0$, which implies that $\Delta p > 0$. ■

Proof of Proposition 6: Notice that the indifference condition (21) is always satisfied when $\gamma = 0$. This is because $u(x) \equiv 1$ when $\gamma = 0$ and the indifference condition becomes trivial.

Let y denote $e^{-\gamma^0}$ so that (21) is

$$y^{p_H - p_L} - q_{LH} - (1 - q_{LH})y^{v_H - v_L} = 0. \tag{24}$$

As γ varies between 0 and $+\infty$, variable y varies between 1 and 0. There is one solution $y = 1$, which corresponds to $\gamma = 0$. In order to prove the Proposition, we need to show that on interval $0 \le y < 1$ there is at most one solution. Given the root $y = 1$, it is equivalent to showing that there are at most two solutions of (24) on $0 \le y \le 1$.

Assume not. If function (24) has three or more roots on $[0, 1]$ then its derivative should have two or more roots on $[0, 1]$. Taking derivative of the LHS of (24) with respect to y and setting it equal to zero we get:

$$(p_H - p_L)y^{p_H - p_L - 1} - (1 - q_{LH})(v_H - v_L)y^{v_H - v_L - 1} = 0,$$

so that

$$1 - (1 - q_{LH})\frac{v_H - v_L}{p_H - p_L}y^{(v_H - v_L) - (p_H - p_L)} = 0.$$

Clearly, the equation above has at most one solution on interval $[0, 1]$. Therefore, equation (24) has at most two solutions on $[0, 1]$. As one solution is $y = 1$ it implies that there is at most one solution when $0 < y < 1$.

To prove the second part of the proposition, we observe that buyers prefer the L-product if

$$1 - e^{-\gamma(v_L - p_L)} \ge q_{LH}(1 - e^{-\gamma(v_L - p_H)}) + (1 - q_{LH})(1 - e^{-\gamma(v_H - p_H)}),$$

which is equivalent to

$$e^{-\gamma(p_H - p_L)} \le q_{LH} + (1 - q_{LH})e^{-\gamma(v_H - v_L)}. \tag{25}$$

When $\gamma = +\infty$ then (25) is satisfied as its LHS is zero, and the RHS is positive. In other words, extremely risk-averse buyers will always purchase the L-product. Thus if $\gamma^0 > 0$ is the solution to (21) then all types with $\gamma < \gamma^0$ purchase the H-product and all types with $\gamma > \gamma^0$ purchase the L-product. ■

Proof of Proposition 7: The proof of the Proposition is fairly long and it consists of two parts. In the first part we reduce the equilibrium system (23) to one equation with one unknown, γ^0. In the second part, we analyze that equation and develop sufficient conditions on $\Phi(\gamma)$ stated in Proposition 7.

We begin the first part of the proof by using Lemma 1 to ignore the last equation in (23) and an unknown variable λ. As in Section 4.2, we will call (23) without the last equation as *the reduced system*. The reduced system has three equations, three unknowns (γ^0, p_L, p_H), and it treats q_{LH} as a given parameter.

The three equations of the reduced system are the indifference condition

$$e^{-\gamma^0(p_H - p_L)} = q_{LH} + (1 - q_{LH})e^{-\gamma^0(v_H - v_L)}, \tag{26}$$

and two FOCs that determine prices:

$$\begin{cases} -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L, p_H)}{\partial p_L}p_L + (1 - \Phi(\gamma^0)) = 0 \\ -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L, p_H)}{\partial p_L}p_H + \Phi(\gamma^0) = 0 \end{cases} \tag{27}$$

Take the second equation of (27) and subtract from it the first equation of (27) we get

$$\frac{\Phi(\gamma^0)}{\phi(\gamma^0)} - \frac{1-\Phi(\gamma^0)}{\phi(\gamma^0)} = \frac{\partial\gamma^0(p_L, p_H)}{\partial p_L}(p_H - p_L).$$

Just as in Section 4.2, we denote $\frac{\Phi(\gamma^0)}{\phi(\gamma^0)} - \frac{1-\Phi(\gamma^0)}{\phi(\gamma^0)}$ as $A(\gamma^0)$. From (26) we get that

$$\begin{aligned}\frac{\partial\gamma^0}{\partial p_L} = -\frac{\partial\gamma^0}{\partial p_H} &= -\frac{\gamma^0 e^{-\gamma^0(p_H-p_L)}}{-(p_H-p_L)e^{-\gamma^0(p_H-p_L)} + (1-q_{LH})(v_H-v_L)e^{-\gamma^0(v_H-v_L)}} = \\ &= \frac{\gamma^0 e^{-\gamma^0(p_H-p_L)}}{(p_H-p_L)e^{-\gamma^0(p_H-p_L)} - (1-q_{LH})(v_H-v_L)e^{-\gamma^0(v_H-v_L)}}.\end{aligned}$$

Let $\Delta p = p_H - p_L$ and $\Delta v = v_H - v_L$. Thus the reduced system becomes a system of two equations and two unknowns:

$$\begin{cases} e^{-\gamma^0\Delta p} &= q_{LH} + (1-q_{LH})e^{-\gamma^0\Delta v} \\ A(\gamma^0) &= \dfrac{\gamma^0 e^{-\gamma^0\Delta p}}{\Delta p e^{-\gamma^0\Delta p} - (1-q_{LH})\Delta v e^{-\gamma^0\Delta v}}\Delta p \end{cases} \tag{28}$$

Next we solve for Δp from the second equation of (28):

$$A(\gamma^0) = \frac{\gamma^0 e^{-\gamma^0\Delta p}}{\Delta p e^{-\gamma^0\Delta p} - (1-q_{LH})\Delta v e^{-\gamma^0\Delta v}}\Delta p.$$

We can re-write it as

$$\begin{aligned} A(\gamma^0)\Delta p e^{-\gamma^0\Delta p} - A(\gamma^0)(1-q_{LH})\Delta v e^{-\gamma^0\Delta v} &= \gamma^0\Delta p e^{-\gamma^0\Delta p} \\ A(\gamma^0)\Delta p - A(\gamma^0)(1-q_{LH})\Delta v e^{-\gamma^0\Delta v} e^{\gamma^0\Delta p} &= \gamma^0\Delta p \\ A(\gamma^0)(1-q)\Delta v e^{-\gamma^0\Delta v}\frac{e^{\gamma^0\Delta p}}{\Delta p} &= A(\gamma^0) - \gamma^0 \\ \frac{\gamma^0 A(\gamma^0)(1-q_{LH})\Delta v e^{-\gamma^0\Delta v}}{A(\gamma^0)-\gamma^0} &= \gamma^0\Delta p e^{-\gamma^0\Delta p}. \end{aligned} \tag{29}$$

Recall from Proposition 5 that $\Delta p > 0$. From second and third equations of (23) then it follows that $\frac{\Phi(\gamma^0)}{\phi(\gamma^0)} > \frac{1-\Phi(\gamma^0)}{\phi(\gamma^0)}$ and, therefore, in equilibrium $A(\gamma^0) > 0$. Furthermore, since $\Delta p > 0$ the LHS of (29) must be positive. In equilibrium $A(\gamma^0) > 0$, which implies that it has to be the case that $A(\gamma^0) > \gamma^0$. If such γ^0 does not exist then (29) cannot be satisfied and no equilibrium with the disclosure of negative information can exist. If, however, such γ^0 does exist, then we can have an equilibrium with the disclosure.

From the indifference condition we get that

$$\gamma^0\Delta p = -\ln(q_{LH} + (1-q_{LH})e^{-\gamma^0\Delta v}),$$

and then

$$\gamma^0\Delta p e^{-\gamma^0\Delta p} = -\ln(q_{LH} + (1-q_{LH})e^{-\gamma^0\Delta v})(q_{LH} + (1-q_{LH})e^{-\gamma^0\Delta v}).$$

Plugging it into (29) we get that the equilibrium value of γ^0 is determined by

$$-\frac{\gamma^0 A(\gamma^0)(1-q_{LH})\Delta v e^{-\gamma^0\Delta v}}{A(\gamma^0)-\gamma^0} = (q_{LH}+(1-q_{LH})e^{-\gamma^0\Delta v})\ln(q_{LH}+(1-q_{LH})e^{-\gamma^0\Delta v}). \tag{30}$$

This completes the first part of the proof. In the second part of the proof, we will analyze equation (30) and derive condition that determine whether the solution exists or not. In what follows we will refer to the RHS and LHS of (30) as simple RHS and LHS without referring to the equation's number.

Proof of i and ii:) As we have established earlier if $A(\gamma) < \gamma$ for every γ then the solution to (29) and, therefore, to (30) does not exist. We will show that this is the case for uniform and convex distributions. The proof is by continuity.

Let the support of $\Phi(\gamma)$ be $[0,\Gamma]$. For uniform and convex distributions it is finite. Inequality $A(\gamma) > \gamma$ is equivalent to $2\Phi(\gamma)-1 > \gamma\phi(\gamma)$. At point $\gamma = 0$ this inequality is violated: $2\Phi(0)-1 = -1$ and $\gamma\phi(\gamma) \geq 0$.[14] At point Γ, the inequality $A(\Gamma) > \Gamma$ is equivalent to $2\Phi(\Gamma) - 1 > \Gamma\phi(\Gamma)$, or $1 > \Gamma\phi(\Gamma)$. It cannot be satisfied. If it is satisfied then $\phi(\Gamma) < 1/\Gamma$. Function ϕ is a (weakly) increasing function and, therefore, $\phi(\gamma) < 1/\Gamma$ for every $\gamma \in [0,\Gamma]$. But then

$$1 = \int_0^\Gamma \phi(s)ds < \int_0^\Gamma \frac{1}{\Gamma}ds = 1,$$

which is a contradiction.

Finally, it cannot be satisfied for intermediate values of γ. Notice that

$$(2\Phi(\gamma)-1)'_\gamma = 2\phi(\gamma) \geq 2\phi(\gamma) - \phi(0) \overset{(!)}{\geq} \phi(\gamma) + \phi'(\gamma)\gamma = (\gamma\phi(\gamma))'_\gamma.$$

The inequality (!) follows from a standard property of convex functions $\phi(\gamma)-\phi(0) \geq \gamma\phi'(\gamma)$, where we added $\phi(\gamma)$ to both sides. We established that $2\Phi(\gamma) - 1$ has greater derivative than $\gamma\phi(\gamma)$. Then if $2\Phi(\gamma) - 1 \geq \gamma\phi(\gamma)$ for at least one $\gamma < \Gamma$ then $2\Phi(\Gamma) - 1 \geq \Gamma\phi(\Gamma)$. It is a contradiction as we already established that $2\Phi(\Gamma) - 1 < \Gamma\phi(\Gamma)$

Proof of iii:) The RHS is a continuous function of γ. It is negative for any $\gamma > 0$. When $\gamma = 0$ it is equal to zero. When $\gamma \to \infty$, its limit is equal to $q_{LH} \cdot \ln(q_{LH}) < 0$.

The LHS is discontinuous when $A(\gamma) = \gamma$. Let $\hat\gamma$ denote the largest root such that $A(\gamma) = \gamma$. We can show that it exists. First, $A(0) = -1/\phi(0) < 0$. Second, $\lim_{\gamma\to\infty}\gamma\phi(\gamma) = 0$. If the limit is positive, say $z > 0$, than it means that for all sufficiently large γ^0, say for all $\gamma > \Gamma^0$, it has to be the case that $\phi(\gamma) > \frac{1}{2}\frac{z}{\gamma}$. But then

$$\int_{\Gamma^0}^\infty \phi(s)ds > \frac{1}{2}\int_{\Gamma^0}^\infty \frac{z}{\gamma}d\gamma = \infty,$$

which is a contradiction since it has to be less or equal than 1. Third,

$$\lim_{\gamma\to\infty}(A(\gamma)-\gamma) = \lim_{\gamma\to\infty}\frac{2\Phi(\gamma)-1-\gamma\phi(\gamma)}{\phi(\gamma)} = \frac{1}{0} = \infty.$$

[14] For convex function $\phi(0)$ is finite.

Given that $A(\gamma) - \gamma$ is continuous we can conclude now that it has roots and that the there is the largest root. In other words, there exists $\hat{\gamma}$ such that $A(\hat{\gamma}) = \hat{\gamma}$ and $A(\gamma) > \gamma$ for every $\gamma > \hat{\gamma}$. Therefore, the LHS is a continuous function for any $\gamma > \hat{\gamma}$.

We can now prove the equilibrium existence. Since $\hat{\gamma}$ is the largest root it means that for any $\gamma > \hat{\gamma}$ it must be the case that $A(\gamma) > \gamma$, and in a sufficiently small right neighborhood of $\hat{\gamma}$ fraction $A(\gamma)/(A(\gamma) - \gamma)$ is close to plus infinity. Then the LHS is close to $-\infty$ and, therefore, is less than the RHS. When γ is close to infinity, the LHS gets arbitrarily close to zero. This is because all terms of the LHS, including $A(\gamma)/(A(\gamma) - \gamma)$, are bounded and the term $e^{-\gamma \Delta v}$ converges to zero. That $A(\gamma)/(A(\gamma) - \gamma)$ is bounded follows from

$$\lim_{\gamma \to \infty} \frac{A(\gamma)}{A(\gamma) - \gamma} = \frac{2\Phi(\gamma) - 1}{2\Phi(\gamma) - 1 - \gamma\phi(\gamma)} = 1.$$

Therefore, for sufficiently large γ the LHS of (30) is less than the RHS. By continuity the solution to (30) exists.

Proof of iv:) Let support of $\Phi(\gamma)$ be $[0, \Gamma]$. Similarly to the convex case earlier (except that all inequalities will get reversed) we can show that $A(\Gamma) > \Gamma$ for a concave cdf. Also, as before, we can show that $A(\gamma) < \gamma$ when γ is sufficiently close to zero. Let $\hat{\gamma}$ be the largest value such that $A(\gamma) = \gamma$. Then the LHS of (30) is continuous when $\gamma \in (\hat{\gamma}, \Gamma]$. As in case iii), one could try to use continuity to establish that the solution to (30) exists. However, it might not work with the original distribution because unless Γ is sufficiently large, the LHS will not be close enough to zero to guarantee that the solution exist.

Consider now a cdf function Φ_α defined as $\Phi(\alpha\gamma)$. It is a concave function with support $[0, \Gamma/\alpha]$. Now the largest value of γ is Γ/α. By taking α sufficiently small we can make the support $[0, \Gamma/\alpha]$ large enough so that $\gamma e^{-\gamma \Delta v}$ and $e^{-\gamma \Delta v}$ can be made sufficiently close to zero within the support.

Term $A(\gamma)/(A(\gamma) - \gamma)$ on the other hand will not change. Let $A_\alpha(\gamma)$ be defined similarly to $A(\gamma)$ but with a cdf Φ_α. Then for any $\gamma \in [0, \Gamma]$,

$$\frac{A_\alpha(\gamma/\alpha)}{A_\alpha(\gamma/\alpha) - \gamma/\alpha} = \frac{A(\gamma)}{A(\gamma) - \gamma}.$$

Indeed,

$$\frac{A_\alpha(\gamma/\alpha)}{A_\alpha(\gamma/\alpha) - \gamma/\alpha} = \frac{2\Phi_\alpha(\gamma/\alpha) - 1}{2\Phi_\alpha(\gamma/\alpha) - 1 - (\gamma/\alpha)\phi_\alpha(\gamma/\alpha)} = \frac{2\Phi(\gamma) - 1}{2\Phi(\gamma) - 1 - (\gamma/\alpha)\alpha\phi(\gamma)} = \frac{A(\gamma)}{A(\gamma) - \gamma}.$$

Thus, when α is sufficiently small we can apply the reasoning of case iii) to function $\Phi_\alpha(\gamma)$ to show that the solution exists. ■

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