Second Mortgages: Valuation and Implications for the Performance of Structured Financial Products

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Abstract

We provide an analytic valuation framework to value second mortgages and first lien mortgages when owners can take out a second lien. We then use the framework to value mortgage-backed securities (MBS) and, in particular, quantify the greater risk associated with MBS backed by first liens that have "silent seconds". Rating securities without accounting for the equity extraction option results in much higher ratings than warranted by expected loss. While the senior tranches rating should be $A1$ rather than Aaa in our benchmark calibration, the big losers from the equity extraction option are the mezzanine tranches who get wiped out.

1 Introduction

During the U.S. housing boom, house prices, as measured by the Case-Shiller Composite-20 index, increased at an average annualized rate of 11% between the first quarter of 2001 and the fourth quarter of 2005. Over this same time period, U.S. homeowners extracted an average of slightly under $700 billion of equity each year relying on cash-out refinancing, home equity lines-of-credit, and second mortgages (Greenspan and Kennedy (2008)). Given the prominent role played by home equity extraction, it is important to understand its implications for the valuation of residential mortgages and, in turn, the properties of structured financial products, like mortgage-backed securities (MBS), collateralized by these mortgages.

To do so, we provide, in the spirit of Black and Cox (1976)), a closed-form structural model to value first liens as well as subordinated mortgages when property owners can take on additional debt by extracting equity from properties that have appreciated. Unlike previous risky mortgage valuation models, we do not exogenously specify property prices and their dynamics. Rather, we take a property's service flow, that is the rent on the property, as our state variable and endogenously derive property prices as well as both senior and junior mortgage values. The role of a property's service flow in our model is analogous to that of a firm's *EBIT* in dynamic capital structure models (see, for example, Goldstein et al. (2001)). To our knowledge, ours is the first analytic model of junior liens.

We then use our structural valuation framework to investigate how the option to take on a second lien affects MBS collateralized by first lien mortgages. We do so because the bursting of the U.S. housing bubble saw the unraveling of many private label MBS (PLMBS). Some observers have argued that these large downgrades reflected the fact that credit rating agencies were blind to the possibility that first lien borrowers could subsequently obtain second loans, so-called "silent seconds" and, as a result, did not recognize the consequences of equity extraction on the performance of MBS. The likelihood that these second loans could have impacted MBS performance is supported by Goodman et al. (2010)'s calculation that more than 50% of first liens in private label securitizations over the 2000 to 2007 time period

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had a second lien behind them, obtained either subsequently as a consequence of second mortgages secured subsequent to origination or simultaneously in the form of piggy-back financing.[1]

To do so, we posit a naïve credit rating agency (CRA) which rates an MBS ignoring the possibility that first lien mortgage borrowers can obtain second liens to extract equity from their appreciated properties. When the resultant MBS structure is confronted by data generated by homeowners who optimally extract equity as well as default, we find that the MBS' resultant performance is broadly consistent with the magnitude of downgrades observed subsequent to the bursting of the U.S. housing bubble. We find that it is the junior tranches that are most heavily affected. In our benchmark calibration, simulations show that that the true expected loss of a Aaa security sized based on a model without equity extraction is four notches lower. However, the mezzanine tranche, which we size to correspond to a $Baa3$ rating (corresponding to $BBB-$ on the S&P rating scale), gets virtually completely wiped out when equity extraction is permitted.

In contrast, our results do not support the argument that the downgrades observed in practice occurred only because the severity of the U.S. housing market downturn was simply underestimated. The distortion in ratings caused by equity extraction is more severe than the difference between *ex ante* ratings and *ex post* losses due to a realized bad aggregate home price scenario. In fact, we find that, absent equity extraction, the senior tranches would have preserved their Aaa ratings even for the aggregate home price paths following the worst origination years. As such, our results suggest that a bad aggregate home price realization, in and of itself, was not enough to generate the losses MBS saw in the financial crisis.

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[1]Piskorski et al. (2015) and Griffin and Maturana (2016) estimate undisclosed seconds *originated at the same time as the first lien* at approximately 10% of loans in PLMBS over roughly the same period. Our concept of equity extraction differs because we model the consequences of extracting equity subsequent to the origination of the first lien and so the estimate in Goodman et al. (2010) corresponds more closely to our model. However, consistent with the regression results in Piskorski et al. (2015) and Griffin and Maturana (2016), the results from our analytic framework confirm that subsequent seconds can affect the performance of MBS.

The plan of this paper is as follows. The next section puts forward and details the properties of a closed-form structural model to value risky residential mortgages. We begin by allowing property owners to only optimally default. In particular, property owners pursue a *static* financing policy in which they rely on an exogenously specified loan-to-value ratio when originally purchasing their property. With subsequent property price appreciation, however, owners cannot extract equity by obtaining a second mortgage. Next we allow owners to optimally extract equity in addition to optimally defaulting. Under a *dynamic* financing policy, owners now get a second lien when property prices appreciate sufficiently. Section 3 investigates the extent to which the unraveling of MBS in the aftermath of the bursting of the U.S. housing bubble can be attributed to naïve credit rating agencies who ignored the presence of second loans. We consider a cash MBS collateralized by a pool of first lien mortgages. The ratings of the MBS are based on the assumption that property owners follow a static financing policy and do not extract equity from their appreciated properties. We demonstrate that if owners actually follow a dynamic financing policy and obtain second mortgages to optimally extract equity, then the presence of these secret seconds can degrade the performance of the first liens so much so that significant downgrades of the MBS result. Section 4 concludes the paper.

2 Closed-Form Valuation of Risky Mortgages

Our underlying state variable is the service flow from a unit of property, denoted by δ, which represents the cost per unit of time of renting the property. The role of δ in our model is analogous to that of a firm's *EBIT* in dynamic capital structure models (see, for example, Goldstein et al. (2001)). This is in contrast to the traditional approach of valuing risky mortgages which takes an unlevered property price as a state variable.[2] Our approach views real estate itself as a contingent claim on δ which can then be valued alongside the risky mortgage. The effects of changing mortgage features on property prices can be easily

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[2]See, for example, Titman and Torous (1989), Kau et al. (1995), and Deng et al. (2000).

explored within this framework.[3]

The dynamics of δ are given by

$$d\delta_t = \delta_t \mu dt + \delta_t \sigma dW_t \qquad (1)$$

and, without loss of generality, we fix $\delta_0 = 1$. Here μ denotes the (instantaneous) drift of the property service-flow process while σ is its (instantaneous) volatility.

We make a number of simplifying assumptions in valuing claims contingent on δ. First, the owner finances an exogenously determined fraction ℓ of the property's purchase price by obtaining an infinite maturity mortgage requiring a fixed coupon payment rate of c. The reliance on mortgage financing reflects, for example, a tax advantage to debt or financing constraints which are not explicitly modeled. The assumption of infinite maturity is for analytic tractability only. Second, we assume the prevailing risk free interest rate, r, is constant. We thus exclude interest rate driven repayments. Finally, the drift of the service-flow process, μ is less than the risk free rate r. Otherwise the value of an infinite stream of service flow will be infinitely large.

2.1 Debt and Equity Without Default or Second Liens

We first consider the case in which the owner can default but cannot take out a second lien. We refer to this case as the static financing policy.

We denote the value of the mortgage by $D(\delta_t)$. The owner's residual claim on the property

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[3]For example, changes in maximum permitted loan-to-value ratios, higher foreclosure costs, the ability of property owners to take out a second lien, the imposition of transaction costs to dissuade second liens, etc.

will be referred to as *equity* and denoted by $E(\delta_t)$. Assuming the owner never defaults then

$$E(\delta_t) = \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}(\delta_s - c)ds\right]$$

$$= \int_t^\infty \left(e^{-r(s-t)}(\mathbb{E}_t[e^{ln\delta_s}] - c)\right)ds = \frac{\delta_t}{r - \mu} - \frac{c}{r} \tag{2a}$$

$$D(\delta_t) = \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}cds\right] = \frac{c}{r}. \tag{2b}$$

In this case, the value of the house is simply the sum of the values of the mortgage and equity

$$D(\delta_t) + E(\delta_t) = \frac{\delta_t}{r - \mu}$$

and corresponds to the value obtained from a simplified version of a user cost of housing model.[4]

2.2   Permitting Default Only

Suppose now that the fixed rate mortgage is contractually defaultable and the property owner cannot take out a second lien. That is, once the property owner chooses the debt-equity mix on the first lien, the amount of debt outstanding cannot be subsequently altered. We will refer to this as a static financing policy. It will serve as a benchmark against the later case of a dynamic financing policy in which property owners can subsequently adjust the amount of debt outstanding to extract equity from their appreciated properties.

Since the mortgage has infinite maturity, we can find $E(\delta)$ and $D(\delta)$ by solving the standard risk-neutral pricing ordinary differential equations (see, for example, Goldstein et al. (2001)). For example, given the dynamics assumed for δ and using Itô's lemma, the

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[4]See, for example, Poterba (1984). The simplification stems from excluding depreciation, taxes, and maintenance costs. The user cost is the cost, including the opportunity cost, that an owner must pay to obtain a unit of housing services.

capital gains are given by

$$dE(\delta_t) = \mu\delta_t E'(\delta_t)dt + \delta_t\sigma E'(\delta_t)dW_t + \frac{1}{2}\sigma^2\delta_t^2 E''(\delta_t)dt$$

while the (instantaneous) dividend rate per unit of time is

$$\delta_t - c.$$

Under risk-neutral pricing, the standard ordinary differential equation (ODE) for equity is thus

$$\frac{1}{2}\sigma^2\delta_t^2 E''(\delta_t) + \mu\delta_t E'(\delta_t) - rE(\delta_t) + \delta_t - c = 0. \qquad (3)$$

The general solutions for equity and debt are given by

$$E(\delta) = e\delta^{x_2} + \frac{\delta}{r - \mu} - \frac{c}{r} \qquad (4a)$$

$$D(\delta) = d\delta^{x_2} + \frac{c}{r}, \qquad (4b)$$

where

$$x_2 = \frac{(\frac{1}{2}\sigma^2 - \mu) - \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2} < 0$$

is the negative root of expression (3)'s associated quadratic equation while e and d are constants to be determined by initial and boundary conditions which characterize this valuation problem. We can exclude the term with a positive power greater than one in the general solutions, expressions (4a) and (4b), because we know that as δ_t approaches infinity they must converge to the corresponding values calculated when default is not permitted, expressions (2a) and (2b).

The initial conditions describing the mortgage and equity at origination, $\delta_0 = \$1$, are

given by

$$D(1) = P \tag{5a}$$

$$E(1) = A - P. \tag{5b}$$

Here P is the mortgage's principal and A is the value at origination of the underlying property financed by the mortgage.

The boundary conditions at the default boundary, $\delta = \delta_B$, are given by

$$E(\delta_B) = 0 \tag{6a}$$

$$E'(\delta_B) = 0 \tag{6b}$$

$$D(\delta_B) = (1 - \alpha)\delta_B A. \tag{6c}$$

The first boundary condition states that at default the property owner's equity stake in the property is worthless. The corresponding smooth-pasting condition is given by the second boundary condition. The final boundary condition captures the fact that at default the lender receives the then prevailing value of the property $\delta_B A$, that is, the property value at origination scaled by the service-flow level at default, all net of foreclosure costs where α is the exogenously specified percentage foreclosure loss.[5]

Because the property is infinitely lived, our valuation framework must make assumptions about its disposition subsequent to a default. In each such case, we assume that foreclosure is immediate and the lender then sells the property for its prevailing value net of foreclosure costs to a buyer who again finances at a loan-to-value ratio of ℓ using a fixed rate infinite maturity mortgage.

Solving the risk-neutral pricing ordinary differential equations subject to these initial and boundary conditions determines the constants e and d as well the default boundary δ_B, the

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[5]Implicit here and throughout this paper is the assumption that mortgage loans are non-recourse thereby limiting a lender's recovery to the property itself.

mortgage principal P and house value at origination A. Finally, the mortgage's fixed coupon payment rate c is implicitly determined by solving

$$\frac{P}{A} = \ell. \tag{7}$$

2.2.1 Solution without Second Liens

We investigate the properties of the model for a base case specification of underlying parameter values. We then sequentially perturb a particular parameter value, holding all other parameter values unchanged, to gauge the model's resultant sensitivities. The results are tabulated in Table 1.

The base case sets the instantaneous drift of the property's service-flow process at $\mu = 2\%$ and an instantaneous volatility of $\sigma = 5\%$ while the prevailing instantaneous risk free rate is fixed at $r = 5\%$. The property owner's desired loan-to-value ratio is assumed to be $\ell = 80\%$ and a foreclosure cost of $\alpha = 25\%$ of the then prevailing property value is incurred in the event of default. A foreclosure discount of $\alpha = 25\%$ is in line with the empirical estimates of the foreclosure discount (see Campbell et al. (2011) and the review of earlier literature in Frame (2010)). In our case, α also captures administrative and legal costs associated with foreclosure.

We solve for the initial value of the home, A, the infinite maturity mortgage's principal, P, as well as its corresponding fixed coupon payment rate c. This results in an implied mortgage rate $y = c/P$. We also compute the level of the service-flow variable which triggers default by the homeowner, δ_B. To gain additional insight into the likelihood of default or, alternatively, the expected length of time until default occurs, we also present the resultant equivalent fixed waiting time to default, $\text{EFWT}(\delta_0)$, as well as the value of an Arrow-Debreu security contingent on default, $\text{ADD}(\delta_0)$, which pays off \$1 only at default.[6]

From Table 1 we see that for the base case parameterization, the initial value of the

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[6]The expected waiting time until default is infinite for a geometric Brownian motion with positive drift ($\mu > 0$). In order to calculate a quantifiable measure of the waiting time until default, we use the value of

property is $A = \$33.28$ while $P = \$26.62$ is borrowed at a mortgage rate of $y = 5.01\%$ to obtain the desired 80% loan-to-value ratio. The property owner subsequently finds it optimal to default when the property's service flow falls from $\delta_0 = \$1$ to $\delta_B = \$0.76$ which gives an equivalent fixed waiting time to default of EFWT = 96 years and an Arrow-Debreu security value contingent on default of ADD = $\$0.008$. Given our parameterization, default is a rare event for a loan-to-value (LTV) of 80% and the resultant default risk raises the cost of borrowing and lowers the property value only slightly.

The initial value of the property A is extremely sensitive to the prevailing risk free rate, r, largely reflecting the fact that it is the discounted value of an infinite stream of service flows. The resultant amount borrowed to maintain the $\ell = 80\%$ loan-to-value ratio varies correspondingly as does the mortgage rate. All else equal, default occurs sooner at a higher risk free rate ($\delta_B = \$0.757$ for $r = 7\%$) as opposed to a lower risk free rate ($\delta_B = \$0.755$ for $r = 3\%$). This reflects the basic property that American options are exercised sooner when

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an Arrow-Debreu security contingent on default defined by

$$\text{ADD}(\delta_t) = \mathbb{E}_t\!\left[e^{-r(\tau_B - t)}\right]$$

where τ_B is the (stochastic) default time. We then define the *equivalent fixed waiting time to default* as the fixed waiting time into the future such that the value of receiving \$1 with certainty after this waiting time would be the same as the value of the Arrow-Debreu security contingent on default. That is, the equivalent fixed waiting time to default, EFWT(δ), satisfies

$$\text{ADD}(\delta) = e^{-r\,\text{EFWT}(\delta)}$$

or

$$\text{EFWT}(\delta) = -\ln(\text{ADD}(\delta))/r.$$

. The general solution for $ADD(\delta)$ is

$$\text{ADD}(\delta) = add_1 \delta^{x_1} + add_2 \delta^{x_2}$$

where x_1 is the positive root. The two value matching conditions are

$$\lim_{\delta\uparrow\infty} ADD(\delta) = 0$$

and

$$ADD_0(\delta_B) = 1.$$

which gives the simple closed form solution

$$ADD(\delta) = \left(\frac{\delta}{\delta_B}\right)^{x_2}.$$

interest rates are higher because the present value of waiting to exercise the option in the future is lower.

As the volatility of the property service flow process increases, the mortgage rate increases. For example, the mortgage rate increases from 5.00% at $\sigma = 3\%$, indicating a nearly riskless mortgage, to 5.09% at $\sigma = 7\%$. Default occurs sooner at a higher volatility but is triggered at a lower value of δ_B reflecting the greater likelihood of a rebound in the property's service flow when volatility is higher. Since foreclosure costs are capitalized in property values, higher foreclosure costs, α, result in a slightly lower initial property value A. A higher LTV, ℓ, means that default will occur sooner, also giving rise to a lower initial property value A and a substantially higher mortgage rate. This feature of the model is in contrast with models emphasizing the role of credit constraints in household behavior as well as empirical evidence.[7] Our model instead predicts lower property values because of the absence of credit constraints. We do not include credit constraints since our interest is in a tractable model of second liens rather than modeling home prices. The effect on property prices of greater default risk can best be seen as highlighting the deadweight costs of default which, in our case, are reflected in home prices.

2.3 Permitting Default and Second Liens

We now permit owners to take out a second mortgage as well as to default.[8] Owners follow a dynamic financing policy allowing them the option to extract equity by increasing their mortgage indebtedness in the event that property prices rise. A new buyer initially obtains a first lien mortgage with an LTV of ℓ_1. The subscript on ℓ indicates the number of extraction options available to the homeowner; 1 means one is left while 0 indicates that there is no

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[7]See, for example Ortalo-Magné and Rady (2006) for a model of how LTVs affect property prices in the presence of credit constraints. Favilukis et al. (forthcoming) presents a quantitative general equilibrium model with credit constraints showing that higher LTVs produce substantially higher equilibrium home prices. Fuster and Zafar (2016) present survey evidence on the effect of LTV restrictions on home prices and review earlier empirical evidence.

[8]In an earlier version of this paper, we considered the case of n junior liens and solved for the case of two junior liens. Since third liens are relatively rare in practice, we confine our analysis to the case of one junior lien. The results with two junior liens are available upon request.

extraction option. To extract equity, the owner obtains a second mortgage in an amount incremental to the previous financing so as to give a *combined* LTV of ℓ_0 given the new higher property value. Like a closed-end second lien in the U.S., this incremental financing is assumed to be junior to all previous financing.[9] We consider the case where $\ell_1 = \ell_0$, such that the owner extracts equity only because property prices have risen as well as the case where $\ell_1 < \ell_0$ which more closely resembles the case of "silent seconds".

Owners, however, do not decrease their mortgage indebtedness if property prices fall. In our framework, the equity holder has no incentive to reduce his debt because of the classic debt overhang problem. As pointed out by Khandani et al. (2013), this "ratchet" effect also reflects the indivisible nature of real estate so that an owner cannot simply reduce leverage by selling a portion of the property and using the proceeds to reduce mortgage indebtedness. Furthermore, mortgage modification is difficult to accomplish in practice.[10]

We assume that an owner can take out a second lien at most one time over the course of owning a property. The owner must determine the service flow at which to optimally take out a second lien. Analogous to the optimal exercise of an American option, the owner trades-off locking in a certain gain from taking a second lien today versus waiting for an even larger gain at some future date. Lenders are aware of the owner's optimal strategy, and price mortgages accordingly.

We solve this problem by dynamic programming. We start by assuming that the owner has already used his second lien option and introduce the opportunity to take out a second lien as we work backwards in time. The owner begins in regime 1 with a first lien mortgage used to purchase a property at an LTV of ℓ_1 and the option to take out a second mortgage remaining. We denote the value of the first lien in regimes 1 and 0 by D_1 and D_{10}. When an owner takes out a second lien, the owner resets the property's combined LTV to ℓ_0 and

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[9]We model second liens as akin to home equity loans for residential properties for tractability but home equity lines of credit (HELOC) play a similar role in our framework. See Agarwal et al. (2006) for an empirical analysis of the difference between the two products.

[10]See, for example, Piskorski et al. (2010), Agarwal et al. (2011), Ghent (2011), Adelino et al. (2013), Mayer et al. (2014), and Ambrose et al. (2016).

enters the next regime, regime 0, with no second lien opportunities remaining. We denote the combined value of the first and second liens in regime 0 by D_0. Given the model's scaling feature, to ease computation and without loss of any generality, we normalize the property's service flow δ to one at the beginning of each regime. The property owner has the option to default in each regime.[11]

To fix matters, assume the owner retains his second lien option. We can take as given the previously obtained first lien. We also take as given the default and extraction triggers, δ_{B_1} and δ_F, as well as the total coupon payment rate c_1 in regime 1. Given the opportunity to take out a second lien, we follow our dynamic programming approach and begin with regime 0 in which the owner cannot extract equity.

Recall, given the process for δ, equation (1), the general solution to the ordinary differential equation governing valuation in our framework is

$$F(\delta) = f_1 \delta^{x_1} + f_2 \delta^{x_2} + \frac{a\delta}{r - \mu} + \frac{b}{r}.$$

Here $x_1 > 1$ and $x_2 < 0$ are the solutions to the quadratic equation

$$\frac{1}{2}\sigma^2 x(x - 1) + \mu x - r = 0.$$

f_1 and f_2 are determined by value matching conditions – two value matching conditions for each value function. We are going to find the value functions for the value of the equity claim in the property,

$$E_0(\delta) = e_{01}\delta^{x_1} + e_{02}\delta^{x_2} + \frac{\delta}{r - \mu} - \frac{c_0}{r}, \tag{8}$$

[11]It is more convenient to work with cumulative as opposed to individual mortgage loans. Firstly, the owner takes out a second lien to achieve a *cumulative* LTV ratio of ℓ_0. Second, the owner only cares about the total coupon payments on the *cumulative* mortgage loans when deciding whether or not to default.

and the value function for debt,

$$D_0(\delta) = d_{01}\delta^{x_1} + d_{02}\delta^{x_2} + \frac{c_0}{r}. \tag{9}$$

Here c_0 denotes the (constant) coupon flow of the debt when there are no opportunities left to extract equity. $D_0(\delta)$ is the value of the *combined* first and second liens.

In the case of no option left to extract equity (i.e., regime 0 only) the value matching conditions for the value of the equity and the debt in order to determine d_{01} and e_{01} are

$$\lim_{\delta\uparrow\infty} D_0(\delta) = \frac{c}{r}$$

$$\lim_{\delta\uparrow\infty}\left(E_0(\delta) - \frac{\delta}{r-\mu}\right) = -\frac{c}{r}.$$

That is, when the service flow gets very high the risk of default is negligible, so the debt value is the value of getting the coupon flow forever. Similarly the equity value is the value of getting the service flow forever and paying the coupon flow for ever. This implies $e_{01} = d_{01} = 0$ such that we can rewrite equations (8) and (9) as

$$E_0(\delta) = e_{02}\delta^{x_2} + \frac{\delta}{r-\mu} - \frac{c_0}{r}, \tag{10}$$

and

$$D_0(\delta) = d_{02}\delta^{x_2} + \frac{c_0}{r}. \tag{11}$$

To determine d_{02} and e_{02}, we look at the value matching conditions at the foreclosure trigger, δ_{B_0}:

$$D_0(\delta_{B_0}) = (1-\alpha)A_1\frac{\delta_{B_0}}{\delta_0} \tag{12}$$

$$E_0(\delta_{B_0}) = 0. \tag{13}$$

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Here A_1 is the value of the property financed by the new owner with an LTV of ℓ_1 with one extraction option remaining, δ_0 is the service flow of the property when initially bought, and α denotes the (proportional) foreclosure costs. Plugging equations (11) and (10) into equations (12) and (13) yields

$$(1 - \alpha)A_1 \frac{\delta_{B_0}}{\delta_0} = d_{02} \delta_{B_0}^{x_2} + \frac{c_0}{r} \tag{14}$$

and

$$0 = e_{02} \delta_{B_0}^{x_2} + \frac{\delta_{B_0}}{r - \mu} - \frac{c_0}{r}. \tag{15}$$

The trigger for when the property owner decides to cease paying the coupon flow to the lender (which then triggers foreclosure immediately) is determined by the smooth pasting condition

$$E_0'(\delta_{B_0}) = 0. \tag{16}$$

Equations (14), (15), and (16) determine δ_{B_0}, e_{02}, and d_{02} for a *given* coupon flow of the debt c_0 and value of the property, A_1.

The coupon flow of the debt is determined when the real estate owner extracts equity at δ_F (this trigger will be determined optimally later, but for now we take it as given). Here the home owner wants to lever up to an LTV of ℓ_0. That is, c_0 is determined as the solution to

$$\frac{D_0(\delta_F)}{D_0(\delta_F) + E_0(\delta_F)} = \ell_0.$$

Having solved the model with no extraction options left, i.e., in regime 0, we turn to solving the model when there is one extraction option left. In this regime (denoted regime 1) the (constant) coupon flow is lower (we will denote it c_1) and therefore the value functions of equity and debt will be different. We will denote them $E_1(\delta)$ and $D_1(\delta)$. More concretely

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we will have

$$E_1(\delta) = e_{11}\delta^{x_1} + e_{12}\delta^{x_2} + \frac{\delta}{r-\mu} - \frac{c_1}{r},$$

and

$$D_1(\delta) = d_{11}\delta^{x_1} + d_{12}\delta^{x_2} + \frac{c_1}{r}.$$

To determine the four constants e_{11}, e_{12}, d_{11} and d_{12}, we need four value matching conditions. Two value matching conditions, one for debt and one for equity, at the foreclosure trigger, δ_{B_1}, and two more value matching conditions at the extraction trigger, δ_F.

At the foreclosure trigger, δ_{B_1}, we have (similarly to the regime 0 case)

$$D_1(\delta_{B_1}) = (1-\alpha)A_1\frac{\delta_{B_1}}{\delta_0}$$

$$E_1(\delta_{B_1}) = 0.$$

In order to determine the trigger itself, δ_{B_1}, we use the smooth pasting condition

$$E_1'(\delta_{B_1}) = 0.$$

At the extraction trigger, δ_F, the property owner takes out the second lien. Thereafter, she needs to service the debt with the new (and higher) coupon flow c_0, but receives the proceeds from the new loan. That is,

$$E_1(\delta_F) = E_0(\delta_F) + (D_0(\delta_F) - D_{10}(\delta_F))$$

We have already derived the value $E_0(\delta_F)$ in regime 0. It reflects the value to the property owner of getting the service flow but paying the higher coupon rate c_0 (and having the non-recourse option of being able to cease coupon payments and walk away from the real estate unit). Similarly, we have already derived the value of *all* the outstanding debt, since $D_0(\delta_F)$ is the value of receiving the coupon flow c_0. Part of this flow, c_1, (note we have

not determined c_1 optimally yet) goes to the senior debt holders (whose value is denoted $D_1(\delta)$ in regime 1) and the rest of the flow $c_0 - c_1$ goes to junior debt holders. That is, the *additional* debt that is issued at the time of equity extraction.

We determine the proceeds from the new loan as the value of the *total* loan minus the value of the existing (now to become senior) loan. The value of the existing loan is the value of receiving the coupon flow c_1 until the home owner defaults. Note that, because she has extracted equity via the additional loan, the property owner needs to service his loans with a service flow c_0 which is greater than c_1. So the default trigger, δ_{B_0}, is higher than δ_{B_1}. When default happens (at the trigger δ_{B_0}) we use the absolute priority rule to determine how the senior and junior loan holders should share the foreclosure proceeds. This means that the senior loan (with coupon flow c_1) will get $\min\{(1-\alpha)A_1\frac{\delta_{B_0}}{\delta_0}, D_1(\delta_0)\}$.

The value function for the senior loan after the junior loan has been issued is denoted D_{10} and has the form

$$D_{10}(\delta) = d_{101}\delta^{x_1} + d_{102}\delta^{x_2} + \frac{c_1}{r}.$$

Now d_{101} is determined by the value matching condition

$$\lim_{\delta\uparrow\infty} D_{10}(\delta) = \frac{c_1}{r}$$

and d_{102} is determined by the value matching condition

$$D_{10}(\delta_{B_0}) = \min\{(1-\alpha)A_1\frac{\delta_{B_0}}{\delta_0}, D_1(\delta_0)\}.$$

Note this means (potentially) that for some parameter values the right hand side of the value matching condition will be $(1-\alpha)A_1\frac{\delta_{B_0}}{\delta_0}$ indicating that senior debt is still risky after the refinancing and that junior debt holders will get nothing at default, whereas for other parameter values, senior debt will be risk free after the refinancing and that there will be some value left for junior debt holders in case of default.

Note also that we value the junior debt residually as the difference between the value of *all* the outstanding debt after the refinancing (which has the coupon flow c_0) and the value of the senior debt (which has the coupon flow $c_1 < c_0$). This is convenient for (at least) two reasons. (i) The property owner only cares about the total service outflow to all debt when she determines when to cease coupon payments, and (ii) the total debt value is independent of the sharing rule at the default trigger.

Finally, we can now determine the value matching condition for debt at the refinancing trigger, δ_F:

$$D_1(\delta_F) = D_{10}(\delta_F).$$

The smooth pasting condition determining the trigger for equity extraction, δ_F, i.e., when the property owner finds it optimal to exercise his one and only (or in case of multiple junior liens 'last') extraction option is

$$E_1'(\delta_F) = \frac{d}{d\delta_F}\left(E_0(\delta_F) + (D_0(\delta_F) - D_{10}(\delta_F)) \right).$$

Recall that c_0 will also be a function of δ_F, since as the property owner waits for a higher service flow level, the additional loan required to achieve the desired LTV of ℓ will be larger. The larger loan amount implies that the service flow, c_0, also increases with δ_F. This, in turn, means that the corresponding foreclosure trigger, δ_{B_0}, is a function of δ_F.

All the calculations up to here are done for *given* values of the coupon flow c_1 and A_1, the value of the property optimally financed with an LTV ratio of ℓ_1 when the service flow level is δ_0 and with one refinancing option.

That is, we determine c_1 at date zero, when we assume an initial service flow level of δ_0, as the solution to

$$\frac{D_1(\delta_0)}{D_1(\delta_0) + E_1(\delta_0)} = \ell_1$$

and we simultaneously determine

$$A_1 = D_1(\delta_0) + E_1(\delta_0).$$

2.3.1 Pricing Properties under the Dynamic Financing Policy with Fixed LTV

Table 2 summarizes the effects of equity extraction for the base case specification of underlying parameter values when $\ell_0 = \ell_1$. For comparison purposes, we also provide corresponding values for the static financing case previously analyzed in which second liens are prohibited. Given the low risk of default with our benchmark calibration, even when the owner can take out a second lien, the pricing properties change little.

However, property values are slightly lower when owners can extract equity. For example, when equity extraction is prohibited, $A = \$33.28$. Permitting owners to extract equity results in a property value of only $A_1 = \$33.25$ at the time the property is acquired (Regime 1), all else being equal.[12] Intuitively, property values are lower in the presence of equity extraction opportunities because the likelihood of future defaults increases and default involves deadweight costs. For example, while the value of an Arrow-Debreu security contingent on default in the absence of equity extraction is ADD = \$0.008, its value given an equity extraction opportunity increases to $\text{ADD}_1 = \$0.012$. The resultant increase in expected foreclosure costs is capitalized in property values.

If property owners can extract equity, the equivalent fixed waiting time to default is slightly shorter as compared to when property owners are prohibited from extracting equity. Given the opportunity to extract equity gives $\text{EFWT}_1 = 88.7$ years, while EFWT $= 96.4$ years in the absence of equity extraction. Equity extraction increases the owner's mortgage indebtedness and so, all else being equal, triggers an earlier default. Similarly, the equivalent fixed waiting time to default increases after the extraction option has been used. For example,

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[12]By way of notation, a variable with a subscript denotes the variable's value when the subscripted number of equity extraction opportunities remain. When a variable is presented without a subscript this corresponds to the case where equity extraction is prohibited.

in our base case parameterization, $\text{EFWT}_1 = 88.7$ years while $\text{EFWT}_0 = 96.4$ years. The reason that default risk is higher in regime 1 (before the borrower exercises the extraction option), is that, for a service flow that begins at $\delta = 1$, the borrower has two ways of defaulting, either at δ_{B_0} or δ_{B_1}. This effect does not exist after exercising the extraction option.

2.3.2 Pricing Properties under the Dynamic Financing Policy with Higher Cumulative LTV with Second Lien

Unlike for case of $\ell_1 = \ell_0$, the EFWT declines after equity extraction when $\ell_0 > \ell_1$, and the probability of default, as measured by the ADD, increases. Importantly, even a modest increase of ℓ_0 to 90%, increases the probability of default by an order of magnitude. In the base case in which we do not permit equity extraction, ADD = \$0.008. When we allow the owner to extract equity up to a 90% LTV, even prior to the point of equity extraction, ADD rises to \$0.049. The increase in the probability of default becomes still more noticable as we increase ℓ_0.

Note that the higher risk to the first lien comes despite property owners not all extracting the equity. In the case of $\ell_0 = 0.95$, the owner does not extract equity until the service flow hits \$1.15. Not surprisingly, higher values of ℓ_0 are associated with higher foreclosure triggers (δ_B) corresponding to earlier foreclosure. At 34 basis points, the spread above the risk free rate for the second lien is still low for $\ell_0 = 0.9$. The spreads rise to 121 and 361 basis points for combined LTVs (CLTVs) of 95% and 98%.

The foreclosure trigger prior to equity extraction $(\delta_B$ in regime 1) falls when the property owner has the option to extract at a higher LTV. The reason is that, by defaulting before extracting his equity, the borrower terminates his equity extraction option. Because this option has value for the borrower, and more value the higher is the CLTV at the equity extraction point, the threshold at which the property owner defaults rises. This is reminiscent of the "competing risks" view of refinancing and default (see, for example, Deng et al. (2000)).

Finally, the fall in the property price for a normalized value of δ of 1 due to the dead-weight costs of foreclosure becomes increasingly noticeable as the maximum CLTV increases. Without equity extraction, the property is worth \$33.28 at origination. For $\ell_0 = 0.95$, the property is worth only \$31.89 at origination of the first lien and still lower for a normalized value of δ of 1 (\$31.65) at equity extraction.

2.3.3   Sensitivity Analysis

Table 3 summarizes the model's sensitivities to changes in its underlying parameters. When we increase the risk free rate of interest, r, the owner exercises his option to extract equity sooner. Focusing on the case in which $\ell_0 = 0.8$, the owner extracts equity at a service flow of $\delta_F = \$1.40$ for $r = 3\%$, but only at a service flow of $\delta_F = \$1.36$ for $r = 7\%$. For the case of $\ell_0 = 0.95$, the extraction boundary falls from \$1.17 to \$1.13 as the risk free rate increases from 3% to 7%. This finding is consistent with the property of American options that pay dividends: exercise becomes earlier as the interest expense increases.

The effect of the risk free rate on the default trigger depends on whether or not the CLTV increases at extraction. For the case of $\ell_1 = \ell_0 = 0.8$, the owner defaults at a lower service flow both before and after extraction for $r = 3\%$ than for $r = 7\%$. This finding is also consistent with the property of a standard American option. When the CLTV at extraction is higher than at the origination of the first lien, after extraction the borrower defaults at a higher service flow as the interest rate increases. However, prior to extraction, a lower value of δ is necessary for the borrower to default when the interest rate is 7% rather than 3%. The reason is that, because a lower value of δ triggers foreclosure after extraction in regime 0, the relative value of waiting is higher as the interest rate increases.

The properties of American options also imply that when the service flow volatility increases, the owner sets trigger points consistent with waiting longer to extract equity and to default. For example, for $\ell_0 = 0.8$ we see that the service flow at which the owner extracts equity when $\sigma = 3\%$ is $\delta_F = \$1.35$, which increases to a trigger service flow of $\delta_F = \$1.43$

when $\sigma = 7\%$. Default, on the other hand, is triggered at a service flow of $\delta_{B_1} = \$0.78$ for $\sigma = 3\%$ but falls to a service flow of $\delta_{B_1} = \$0.73$ for $\sigma = 7\%$. The equivalent fixed waiting times to default are shorter in the presence of more volatile housing service flows.

The extraction boundary also varies with foreclosure costs, α. However, for our base case choices of the other parameters, there is relatively litle change. Higher foreclosure costs result in lower property values because of the greater deadweight costs. Similarly, borrowers face higher interest rates as foreclosure costs rise. However, the extraction boundary rises as foreclosure costs rise. For example, for $\ell_0 = 0.8$ the owner extracts equity at a service flow of $\delta_F = \$1.30$ when $\alpha = 20\%$ but, for foreclosure costs of $\alpha = 30\%$, equity extraction is triggered much later at a higher service flow of $\delta_F = \$1.47$.

3 "Silent Seconds" and the Unraveling of MBS

The bursting of the U.S. housing bubble saw the unraveling of many MBS. For example, Ghent et al. (2016) find that by summer 2013, over a third of AAA private label MBS (PLMBS) originated 1999-2007 were in default while more than 80% of PLMBS securities originally rated investment grade but below AAA were in default.

Some critics have argued that these large downgrades reflected the fact that credit rating agencies (CRAs) simply underestimated the severity of the U.S. housing market downturn which caused a sharp increase both in the level of defaults as well as in the correlation of defaults across homeowners. Others have suggested that CRAs were blind to the fact that first lien borrowers could subsequently obtain second loans and, as a result, ignored the consequences of equity extraction on the performance of MBS.[13] These so-called "silent seconds" increased the likelihood that a homeowner would default in the event of a downturn in house prices. Moreover, the fact that so many U.S. homeowners relied on second mortgages to extract equity from their homes during the run-up in house prices through 2006 meant that they were more likely to default *en masse* when house prices subsequently fell.

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[13]See, for example, the discussion in Lewis (2010), page 100.

We now investigate the extent to which the unraveling of MBS in the aftermath of the bursting of the U.S. housing bubble can be attributed to CRAs ignoring borrowers' potential to take out a second lien. We also shed light on the role of CRAs underestimating the severity of the U.S. housing downturn on the subsequent performance of MBS.

3.1   A Hypothetical Cash MBS

We consider an originator who only originates first lien mortgages. In particular, we assume that at date t the lender has originated 1,000 first lien mortgages. Consistent with our valuation framework, each mortgage is an infinite-maturity loan characterized by the base case LTV of $\ell = 80\%$ and foreclosure costs of $\alpha = 25\%$. Each underlying property's service flow is (instantaneously) log normally distributed with the base case (instantaneous) drift of $\mu = 2\%$ and volatility of $\sigma = 5\%$. To model correlation between the underlying properties, we split the service-flow into two components: a common component shared across all properties and a property-specific component. The common component has 40% of the volatility of the idiosyncratic component. However, the common component comprises 60% of the process. This calibration is consistent with the common share of home price variance estimated by Goetzmann (1993). Finally, the risk free rate of interest is fixed at $r = 5\%$.

At date t the loan originator deposits the 1,000 first lien mortgages in a trust and receives, in turn, the prevailing value of the loans. Relying on this pool of first lien mortgages as collateral, the trust issues an MBS consisting of two interest-bearing certificates, one senior and the other mezzanine, together with a non-interest bearing residual claim on the mortgage pool's cash flows. The interest rate owed on the certificates is the risk free rate plus the expected loss on the certificate. We assume the MBS has a maturity of 10 years.

MBS prioritize payments to their constituent securities. In our case, the first priority is interest payments to the senior certificate. The second priority is interest payments to the mezzanine certificate. These interest payments are paid currently. Next are principal payments to the senior certificate, followed by principal payments to the mezzanine certificate.

Any remaining cash flows are then allocated to the residual certificate. Principal payments are paid on an accrued basis on the MBS' finite maturity date. This payout convention is required because we assume infinite-maturity mortgages are backing a finite maturity MBS.

If a default occurs, we assume the underlying property is immediately sold in a foreclosure sale and the resultant sale proceeds, net of administrative costs, are deposited by the trust in a risk free rate bearing account.[14] Losses are allocated first to the residual class, then to the mezzanine certificate, and finally to the senior certificate.

At the MBS' maturity, the trustee sells the first lien mortgages remaining in the pool at their prevailing marketprices. The trustee uses these proceeds together with the liquidation of any accounts in the trust arising from previous foreclosures to make principal payments according to the MBS' priority structure. The trust is then terminated.

3.2   Sizing MBS

Apart from subordination, we assume that the MBS deal has no other credit enhancements. Therefore the credit rating assigned to a particular certificate depends solely on the degree of protection afforded the certificate by other certificates subordinate to it. The more subordination provided a particular certificate, the smaller the certificate's expected losses and so the higher its credit rating. Prior to the foreclosure crisis, Moody's, for example, assigned ratings for both corporate bonds and structured products based on the "idealized expected loss rates" given in Table 4. We rely on these loss rates in determining the ratings assigned to the interest-bearing certificates of our hypothetical MBS. Our loss rates include both loss of principal and loss of interest although, given the waterfall we specify, the vast majority of the losses are lost principal. As in practice, the residual certificate is not rated.[15]

To attain a particular credit rating requires us to determine the size of a certificate's

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[14]We assume that the MBS' pooling and servicing agreement does not require the replacement of any defaulted loan in the pool regardless of how soon the default occurs.

[15]Post-financial crisis, some of the rating agencies, including Moody's, have issued separate scales for rating structured finance securities such as MBS. See Cornaggia et al. (2015) for a discussion of the challenges of using the same scales for rating across asset classes.

principal so that the desired level of expected losses can be achieved given the underlying collateral's risk characteristics. To do so, we first increase the fraction of the MBS' principal allocated to the senior certificate until across all of our simulations of the underlying correlated collateral the resultant fraction experiences an average loss rate equal to that allowed by the senior certificate's desired rating, for example, *Aaa*. Given we have sized the senior certificate, we then proceed in a similar fashion to size the mezzanine certificate so that its fraction has an average loss rate across all of our simulations equaling that allowed by its desired rating, for example, *Baa*3. The remaining fraction of the MBS' principal is then allocated to the residual certificate. While it is the structurer of the financial institution issuing the security that officially sizes the certificates of an MBS, prior to the financial crisis, structurers frequently consulted with the CRAs regarding what deal features were necessary for a certain portion of the deal to receive particular ratings. Hereafter, we thus sometimes refer to the CRA as being the institution to size the tranches.

3.3   Simulation Results

We first assume that the CRA is naïve meaning that, when rating the MBS, it does not allow for the possibility that first lien borrowers may subsequently extract equity. To emulate this naïve CRA, we simulate, through the MBS' maturity date, the correlated service-flow processes underlying each first lien mortgage included in the pool. We assume that the loan originators price the first liens assuming that property owners cannot extract equity. Relying on our static financing policy framework, in which owners cannot extract equity but optimally default, we then calculate the losses incurred across the pool for each simulation when property owners can extract equity by going up to a 95% LTV. We repeat this simulation exercise 1,000,000 times and size the MBS so that the naïve CRA rates the senior certificate as *Aaa* and the mezzanine certificate as *Baa*3.

Table 5 shows that, for the assumed base case parameters, the senior certificate accounts for approximately 97% of the MBS' principal while the mezzanine certificate's size is ap-

proximately 3% with only a tiny residual. While equity extraction adversely affects both tranches, the mezzanine tranche is far more affected than the senior tranche. The true rating of the senior tranche with equity extraction is actually $A1$, a full notch lower than it should be, because the losses are 80 times what the CRAs tolerate for a AAA security. The mezzanine tranche is almost completely wiped out, losing essentially all of its value, by failing to account for the property owner's option to extract equity. Rather than being in the investment grade category of $Baa3$, the actual rating of the mezzanine tranche is four subnotches lower at Ca.

An alternative way of understanding the erroneous ratings is to resize the tranches under the assumption that the CRAs understand the borrower's option to extract equity. The bottom panel of Table 5 shows the size of the tranches if the rating agency takes into account the borrower's option to extract equity. The AAA tranche is only 94% of the deal such that it has double the subordination, an additional three percentage points. The biggest difference is for the mezzanine tranche though. While it has nearly no subordination under the naïve CRA, such that it gets almost completely wiped out, it gets 4 percentage points of subordination when the CRA takes into account the extraction option. In essence, ignoring equity extraction makes the mezzanine tranche akin to a residual rather than an investment-grade security.

These calculations assume that the homeowners in the pool are confronted with a wide variety of house price paths across the 1,000,000 simulations. We can also determine the losses incurred by homeowners in the pool, and therefore the losses passed on to the MBS investors, if house prices behaved similarly to the actual path that U.S. house prices followed after origination. To do so, we measure U.S. home prices by the monthly $FHFA$ purchase only non-seasonally adjusted repeat sales index for the U.S. For purposes of our subsequent analysis, we consider MBS issuance dates of 2004, 2005, 2006, and 2007. To investigate the performance of this MBS over the actual path of U.S. home prices, we now restrict our attention to scenarios in which the common component of the house price process follows

the actual monthly $FHFA$ index beginning in January 2004, January 2005, January 2006, or January 2007.[16] To begin with, we calculate losses across these particular paths assuming that investors optimally default but cannot extract equity. This allows us to determine what losses the MBS would have incurred due solely to the adverse realization of U.S. house prices.

In Table 5, we see that relative to their original ratings, the senior certificate would remain unaffected by the path of home prices if homeowners could not extract equity. Figure 1 shows that the losses from equity extraction are far greater for both tranches than those that would result solely from an unfortunate realization of aggregate home prices. Absent equity extraction, in fact, mezzanine investors in 2004 MBS would fare even better than the stated rating as the rating corresponding to the actual loss experience corresponds to a rating of $Aa2$. Mezzanine investors in 2005, 2006, and 2007 fare worse with 2007 being the worst year. However, it is only once extraction is permitted that the mezzanine tranche is wiped out for every origination year.

Table 6 investigates the sensitivity of the effect of equity extraction to changes in the assumed underlying parameters. As before, given a particular set of parameters, the naïve CRA sizes the MBS so that the senior certificate is Aaa rated and the mezzanine certificate is $Baa3$ rated. As before, the naïve CRA assumes homeowners optimally default but do not extract equity, and relies on 1,000,000 simulated house price paths to assess housing's risk characteristics. We then take the given MBS and recalculate each certificate's expected losses assuming that homeowners can extract equity as well as default over all 1,000,000 simulated house price paths.

Notice that, as compared to the base case, the size of the Aaa rated senior certificate decreases as the riskiness of the underlying collateral increases. In other words, the naïve CRA requires more subordination for the senior certificate to achieve a Aaa rating when the collateral's risk increases. For example, for a service flow volatility of only 3%, all else being

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[16]For the last year of the simulation for the 2007 historical experience, we revert to our base case assumption for the common component since there is not historical home price data available for 2017 as of the writing of this paper.

equal, the size of the *Aaa* rated senior certificate is almost 100% of the MBS' principal and there is no mezzanine tranche. When the service flow volatility rises to 7%, only 85% of the MBS' principal is rated *AAA*. In addition, the naïve credit rating agency requires more subordination in order for the senior certificate to be *Aaa* rated if interest rates are high and when foreclosure costs are high.

When we calculate expected losses across all 1,000,000 simulated house price paths assuming homeowners optimally extract equity as well as optimally default, the senior certificate is downgraded by up to six notches for $\ell_0 = 0.95$, while the mezzanine certificate is usually wiped out. The smallest downgrades correspond to the case in which volatility is high or foreclosure costs are high. The reason is that it is only in these scenarios where there is substantive cushioning from the residual tranche. The worst outcome for the senior tranche when we hold ℓ_0 fixed at 95% is actually for the seemingly safe scenario in which the volatility of the service flow is only 3% since this is the case in which there is a miniscule residual and actually no mezzanine tranche. Not surprisingly, the largest downgrades result when property owners can go up to a CLTV of 98%. Here the senior certificate would be downgraded to the non-investment grade *Ba2* while the mezzanine tranche remains completely wiped out.

4   Summary and Conclusions

Given the prominent role played by junior liens during the recent run-up in U.S. house prices, this paper explores the implications for the pricing and properties of residential mortgages, both first liens as well as junior liens, and, in turn, structured financial products based on the first lien mortgages with junior liens behind them. We find that equity extraction is a necessary condition to generate the magnitude of losses we observed during the crisis on the senior tranches. Bad realizations of aggregate home prices are not enough in and of themselves enough to impair the senior tranches. Nevertheless, we find that the mezzanine

tranches are far more affected by the presence of silent seconds than the senior tranches. Our work suggests that the potential to take on a junior lien subsequent to origination of the first lien should be taken into account when pricing first mortgages and, especially, when structuring MBS.

Appendix: The Model with n Extraction Options

In this Appendix we detail the corresponding initial conditions as well as value-matching and smooth-pasting conditions characterizing the property owner's optimal default and equity extraction decisions for the general case in which the owner has n extraction options.

Assume the owner is in regime j in which j of the original n cash-out refinancing options remain. This means that the owner has already cash-out refinanced at each of the previous regimes $i = j + 1, \ldots, n$. At the beginning of regime j we have the initial conditions:

$$D_{jj}(1) = P_j$$

$$E_j(1) = A_j - P_j$$

where P_j denotes the cumulative principal borrowed after the owner's jth refinancing and A_j denotes the then prevailing value of the underlying property. The total coupon payment rate the owner will pay during regime j, denoted c_j, is determined so that

$$\frac{P_j}{A_j} = \ell.$$

The default value-matching and smooth-pasting conditions in regime j are given by

$$E_j(\delta_{B_j}) = 0$$

$$E_j'(\delta_{B_j}) = 0$$

$$D_{ij}(\delta_{B_j} \prod_{k=j+1}^{i} \delta_{F_k}) = \min\left\{(1-\alpha)A_n\delta_{B_j} \prod_{k=j+1}^{i} \delta_{F_k}, \frac{c_i}{r}\right\}$$

for $i = j, \ldots, n$. The homeowner defaults when the house's service flow is sufficiently low relative to the total coupon payment rate, c_j, to all the mortgage loans issued. In the event of default, the homeowner defaults on all mortgages and lenders are assumed to foreclose instantaneously thereafter and allocate the available proceeds amongst the existing liens

according to absolute priority. To keep track of this, we have $n - j + 1$ value-matching conditions for the cumulative mortgage values. In particular, cumulatively all the mortgages issued in all regimes up to and including regime j, this value being denoted by D_{jj}, will receive $(1 - \alpha)A_n\delta_{B_j}$ in case of default. This reflects the fact that the creditors receive the property value net of foreclosure costs, α, and that the property can be sold to a new homeowner who again will have exactly n refinancing options.

Similarly, for $j \geqslant 1$, the refinancing value-matching and smooth-pasting conditions in regime j are given by[17]

$$E_j(\delta_{F_j}) = \delta_{F_j}A_{j-1} - D_{j,j-1}(\delta_{F_j})$$

$$E_j'(\delta_{F_j}) = A_{j-1} - D_{j,j-1}'(\delta_{F_j})$$

$$D_{ij}(\delta_{F_j}\prod_{k=j+1}^{i}\delta_{F_k}) = D_{i,j-1}(\delta_{F_j}\prod_{k=j+1}^{i}\delta_{F_k})$$

for $i = j, \ldots, n$.

Since D_{ij} is the *cumulative* value of all the mortgages issued to the homeowner in regime i and all previous regimes (with higher indices, $i + 1, \ldots, n$), we can determine the value (as of regime j) of *just* the mortgage issued in regime i by calculating

$$D_{ij}(\delta_i) - \frac{1}{\delta_{F_{i+1}}}D_{i+1,j}(\delta_{F_{i+1}}\delta_i)$$

for $i = 0, \ldots, n - 1$ and $j = 0, \ldots, i$. Similarly, the coupon payment rate of the mortgage *just* issued in regime i is calculated as

$$c_i - \frac{c_{i+1}}{\delta_{F_{i+1}}},$$

for $i = 0, \ldots, n - 1$.

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[17]Note that for the case $j = 0$ there are no cash-out refinancing opportunities remaining and so these value-matching and smooth-pasting conditions do not apply.

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This table provides values of the underlying property (A), first lien mortgage principal (P) and mortgage rate (y) in addition to the critical service flows (δ_B) at which the homeowner optimally defaults with corresponding equivalent fixed waiting time to default (EFWT) and values of Arrow-Debreu security contingent on default (ADD). We assume a base case of parameter values as well as perturbing the base case by assuming an alternative parameter value as indicated in the Table's column headings. The Base Case assumes that the risk free rate, r, is 0.05, the volatility of the property service flow is 5%, the foreclosure discount, α, is 25%, and the LTV at origination, ℓ is 80%.

| | Base Case | r | | σ | | α | | ℓ | |
		3%	7%	3%	7%	20%	30%	70%	90%
A	$ 33.28	$ 99.81	$ 19.97	$ 33.33	$ 33.01	$ 33.29	$ 33.27	$ 33.33	$ 32.83
P	$ 26.62	$ 79.85	$ 15.98	$ 26.67	$ 26.41	$ 26.63	$ 26.62	$ 23.33	$ 22.98
y	5.01%	3.01%	7.01%	5.00%	5.09%	5.01%	5.01%	5.00%	6.56%
delta_B	$ 0.757	$ 0.755	$ 0.759	$ 0.783	$ 0.728	$ 0.757	$ 0.757	$ 0.662	$ 0.855
ADD	$ 0.008	$ 0.010	$ 0.007	$ 0.000	$ 0.051	$ 0.008	$ 0.008	$ 0.001	$ 0.067
EFWT	96	154	71	224	59	96	96	143	54

Table 2: Valuation with Equity Extraction: Base Case Parameterization

This table provides values of the underlying property (A), second lien mortgage rate (y) and cumulative mortgage rate ($\bar{y}$) when the property owner can optimally extract equity. We assume the base case of parameter values (r=0.05, $\mu = 0.02$, $\sigma = 0.15$, $\alpha = 0.25$, and $\ell_1 = 0.8$). The critical service flows at which the owner optimally extracts equity (δ_F) and optimally defaults (δ_B as a percentage of service flow at purchase or equity extraction) with corresponding equivalent fixed waiting times to default (EFWT) and values of Arrow-Debreu security contingent on default (ADD) are also provided. Regime 1 refers to the period prior to equity extraction and regime 0 refers to the period after equity extraction.

	A	P	y	$\bar{y}$	δ_B	δ_F	ADD	EFWT
	No Equity Extraction Allowed							
	$ 33.28	$ 26.62	5.012%	5.012%	$ 0.757		$ 0.008	96.4
	With Equity Extraction Option and $\ell_0 = \ell_1 = 0.8$							
Regime 1	$ 33.25	$ 26.60	5.012%	5.012%	$ 0.757	$ 1.38	$ 0.012	88.7
Regime 0	$ 33.28	$ 26.63	5.011%	5.012%	$ 0.757		$ 0.008	96.4
	With Equity Extraction Option and $\ell_0 = 0.9, \ell_1 = 0.8$							
Regime 1	$ 32.90	$ 26.32	5.016%	5.016%	$ 0.755	$ 1.22	$ 0.049	60.1
Regime 0	$ 32.84	$ 29.55	5.335%	5.102%	$ 0.855		$ 0.067	54.1
	With Equity Extraction Option and $\ell_0 = 0.95, \ell_1 = 0.8$							
Regime 1	$ 31.89	$ 25.51	5.028%	5.028%	$ 0.749	$ 1.15	$ 0.149	38.1
Regime 0	$ 31.65	$ 30.07	6.213%	5.337%	$ 0.910		$ 0.196	32.5
	With Equity Extraction Option and $\ell_0 = 0.98, \ell_1 = 0.8$							
Regime 1	$ 29.35	$ 23.48	5.063%	5.063%	$ 0.732	$ 1.10	$ 0.338	21.7
Regime 0	$ 28.85	$ 28.28	8.606%	5.927%	$ 0.951		$ 0.416	17.5

Table 3: Valuation with Equity Extraction: Comparative Statics

		A	P	y	$\bar{y}$	δ_B	δ_F	ADD	EFWT
					r=3%				
$\ell_1 = 0.8$	Regime 1	$ 99.66	$ 79.72	3.009%	3.009%	$0.755	$ 1.40	$ 0.016	139
	Regime 0	$ 99.81	$ 79.85	3.009%	3.009%	$0.755		$ 0.010	154
$\ell_0 = 0.95$	Regime 1	$ 94.75	$ 75.80	3.013%	3.013%	$0.751	$ 1.17	$ 0.174	58
	Regime 0	$ 94.53	$ 89.80	3.767%	3.221%	$0.909		$ 0.208	52
					r=7%				
$\ell_1 = 0.8$	Regime 1	$ 19.96	$ 15.97	7.014%	7.014%	$0.758	$ 1.36	$ 0.009	67
	Regime 0	$ 19.97	$ 15.98	7.012%	7.014%	$0.759		$ 0.007	71
$\ell_0 = 0.95$	Regime 1	$ 19.25	$ 15.40	7.043%	7.043%	$0.749	$ 1.13	$ 0.131	29
	Regime 0	$ 19.05	$ 18.10	8.624%	7.438%	$0.911		$ 0.187	24
					$\sigma = 3\%$				
$\ell_1 = 0.8$	Regime 1	$ 33.33	$ 26.67	5.000%	5.000%	$0.783	$ 1.35	$ 0.000	217
	Regime 0	$ 33.33	$ 26.67	5.000%	5.000%	$0.783		$ 0.000	224
$\ell_0 = 0.95$	Regime 1	$ 33.08	$ 26.47	5.001%	5.001%	$0.781	$ 1.13	$ 0.028	72
	Regime 0	$ 33.04	$ 31.38	5.196%	5.051%	$0.931		$ 0.037	66
					$\sigma = 7\%$				
$\ell_1 = 0.8$	Regime 1	$ 32.81	$ 26.25	5.089%	5.089%	$0.727	$ 1.43	$ 0.073	52
	Regime 0	$ 33.01	$ 26.40	5.084%	5.087%	$0.728		$ 0.052	59
$\ell_0 = 0.95$	Regime 1	$ 30.41	$ 24.33	5.123%	5.123%	$0.714	$ 1.17	$ 0.278	26
	Regime 0	$ 30.10	$ 28.59	7.471%	5.758%	$0.892		$ 0.345	21
					$\alpha = 20\%$				
$\ell_1 = 0.8$	Regime 1	$ 33.26	$ 26.61	5.010%	5.010%	$0.757	$ 1.30	$ 0.012	88
	Regime 0	$ 33.29	$ 26.63	5.009%	5.010%	$0.757		$ 0.008	96
$\ell_0 = 0.95$	Regime 1	$ 32.08	$ 25.67	5.023%	5.023%	$0.750	$ 1.08	$ 0.167	36
	Regime 0	$ 31.98	$ 30.38	6.190%	5.279%	$0.910		$ 0.194	33
					$\alpha = 30\%$				
$\ell_1 = 0.8$	Regime 1	$ 33.24	$ 26.59	5.014%	5.014%	$0.757	$ 1.47	$ 0.011	90
	Regime 0	$ 33.27	$ 26.62	5.013%	5.014%	$0.757		$ 0.008	96
$\ell_0 = 0.95$	Regime 1	$ 31.74	$ 25.39	5.032%	5.032%	$0.748	$ 1.22	$ 0.131	41
	Regime 0	$ 31.32	$ 29.76	6.241%	5.396%	$0.911		$ 0.199	32

Table 4: Moody's Ratings for Corporate Bonds and Their Expected Loss Criteria

This table shows Moody's ratings for corporate bonds and their corresponding expected loss rates. Expected loss rates are over a four-year horizon.

Corporate Rating	Expected Loss Rate
Aaa	0.0010%
Aa1	0.0116%
Aa2	0.0259%
Aa3	0.0556%
A1	0.1040%
A2	0.1898%
A3	0.2870%
Baa1	0.4565%
Baa2	0.6600%
Baa3	1.3090%
Ba1	2.3100%
Ba2	3.7400%
Ba3	5.3845%
B1	7.6175%
B2	9.9715%
B3	13.2220%
Caa1	17.8634%
Caa2	24.1340%
Caa3	36.4331%
Ca	50.0000%
C	80.0000%
D	90.0000%

Table 5: MBS and "Silent Second": Base Case Parameters

We size a cash MBS and determine its certificates' expected losses under a variety of assumptions. We assume the base case of parameter values. In Panel A, in the Baseline case, a naïve credit rating agency relies on 1,000,000 simulation paths in which homeowners optimally default but cannot extract equity. Originators price the first liens assuming property owners cannot extract equity. In the "U.S. Experience", the common component of home prices follows the behavior of U.S. house prices since origination. Loss rates are normalized to correspond to a four year horizon shown in Table 4. Panel B shows the correct sizing of the tranches when the credit rating agency takes into account the borrowers' ability to extract equity.

| Panel A: Naïve Credit Rating Agency | | | All Paths | | U.S. Experience | | | |
| | Baseline | | Extraction Permitted | | No Extraction | | Extraction Permitted | |
Size	Loss Rate	Rating	Loss Rate	Rating	Loss Rate	Rating	Loss Rate	Rating
2004 Originations:								
0.968	0.001%	Aaa	0.080%	A1	0.000%	Aaa	0.975%	Baa3
0.031	1.309%	Baa3	64.831%	C	0.014%	Aa2	99.997%	D
2005 Originations:								
0.968	0.001%	Aaa	0.080%	A1	0.000%	Aaa	0.695%	Baa3
0.031	1.309%	Baa3	64.831%	C	2.459%	Ba2	99.997%	D
2006 Originations:								
0.968	0.001%	Aaa	0.080%	A1	0.000%	Aaa	0.276%	A3
0.031	1.309%	Baa3	64.831%	C	15.026%	Caa1	99.822%	D
2007 Originations:								
0.968	0.001%	Aaa	0.080%	A1	0.000%	Aaa	0.233%	A3
0.031	1.309%	Baa3	64.831%	C	17.958%	Caa2	98.383%	D

Panel B: Saavy Credit Rating Agency (Correct Sizing if Extraction Permitted)				
		All Paths		
		No Extraction		Extraction Permitted
Size	Loss Rate	Rating	Loss Rate	Rating
0.941	0.000%	Aaa	0.001%	Aaa
0.021	0.020%	Aa2	1.309%	Baa3

Table 6: MBS and "Silent Seconds": Comparative Statics

We size a cash MBS and determine its certificates' expected losses under a variety of assumptions. We perturb the base case of parameter values by assuming an alternative parameter value as indicated. A naïve credit rating agency relies on 1,000,000 simulation paths in which homeowners optimally default but cannot extract equity. Originators price the first liens assuming property owners cannot extract equity. Loss rates are normalized to correspond to the four year horizon shown in Table 4.

| | Baseline | | All Paths with Refis | |
Size	Loss Rate	Rating	Loss Rate	Rating
		$r = 3\%$		
0.970	0.001%	Aaa	0.012%	Aa2
0.030	1.309%	Baa3	37.068%	Ca
		$r = 7\%$		
0.964	0.001%	Aaa	0.286%	A3
0.034	1.309%	Baa3	89.451%	D
		$\sigma = 3\%$		
0.999	0.001%	Aaa	0.202%	A3
—	1.309%	Baa3	—%	—
		$\sigma = 7\%$		
0.847	0.001%	Aaa	0.005%	Aa1
0.138	1.309%	Baa3	14.922%	Caa1
		$\alpha = 20\%$		
0.974	0.001%	Aaa	0.157%	A2
0.025	1.309%	Baa3	75.759%	C
		$\alpha = 30\%$		
0.960	0.001%	Aaa	0.020%	Aa2
0.038	1.309%	Baa3	51.33%	C
		$\ell_0 = 80\%$		
0.968	0.001%	Aaa	0.00103%	Aa1
0.031	1.309%	Baa3	1.140%	Baa3
		$\ell_0 = 90\%$		
0.968	0.001%	Aaa	0.003%	Aa1
0.031	1.309%	Baa3	7.498%	B1
		$\ell_0 = 98\%$		
0.968	0.001%	Aaa	3.365%	Ba2
0.031	1.309%	Baa3	99.997%	Ca

Figure 1: Losses from Equity Extraction

The first two bars of each figure (corresponding to "All Simulated") show the losses, normalized to correspond to a 4-year horizon for all simulated paths. We size the *Aaa* tranche to have a loss of 0.001% under the assumption of no extraction. We size the *Baa*3 tranche to have a loss of 1.3%. These losses are those shown in the first bar. The last 8 bars show the losses without and with the equity extraction option when the common component of home prices follows the FHFA national index from January of the year of origination shown.