Retail competition and cooperative advertising

Xiuli He [a], Anand Krishnamoorthy [b], Ashutosh Prasad [c], Suresh P. Sethi [c,*]

[a] Belk College of Business, Univerisy of North Carolina at Charlotte, NC 28223, United States
[b] College of Business Administration, University of Central Florida, Orlando, FL 32816, United States
[c] School of Management, University of Texas at Dallas, Richardson, TX 75080, United States

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ABSTRACT

We consider a cooperative advertising channel consisting of a manufacturer selling its product through a retailer in competition with another independent retailer. The manufacturer subsidizes its retailer's advertising only when a certain threshold is positive. Moreover, the manufacturer's support for its retailer is higher under competition than in its absence.

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1. Introduction

In the context of manufacturer–retailer relationships, the decisions of each channel member affect the other's profitability and strategy choices. For example, a retailer advertises a manufacturer's product to increase its sales, but it may not do so to the extent that the manufacturer might prefer. As a result, the manufacturer might provide incentives to the retailer. The situation is complicated even further if there is more than one retailer competing in the retail market with a common customer base. This case is not uncommon in situations where territories are not exclusive. The manufacturer and the retailers take actions that try to maximize their individual profits.

Manufacturers often use cooperative advertising to influence their retailers' advertising decisions. Cooperative advertising is an arrangement whereby a manufacturer agrees to reimburse a portion of the advertising expenditures incurred by retailers for selling its product [2]. This is often preferred by the manufacturer to doing local advertising by itself because of the expertise in the local markets that it would need but often does not have; the retailers usually have much better local knowledge about customers.

Cooperative advertising programs are common, and can be a significant part of the advertising budgets of manufacturers. By some estimates, more than $25 billion was spent on cooperative advertising in 2007, compared to $15 billion in 2000 and $900 million in 1970, and approximately 25%–40% of all manufacturers used this arrangement [6,23]. These figures underlie the importance of understanding cooperative advertising programs in manufacturer–retailer relationships to be able to improve their effectiveness.

In order to deepen our understanding of cooperative advertising programs given their increasing popularity, we model a dynamic distribution channel in which a manufacturer sells a product through a retailer who is competing with another independent retailer. The retailers choose their advertising efforts after the manufacturer decides the extent of its support for its retailer's advertising activity. This is called "subsidy rate", i.e., the portion of the retailer's advertising expenditures that the manufacturer will subsidize. The sequence of events is as follows: The manufacturer first chooses its subsidy rate. Taking this subsidy rate into consideration, the two retailers simultaneously choose their local advertising levels. Sales are then realized based on their advertising efforts.

The inter-temporal effects of advertising require the use of dynamic models; see [11] for a survey of dynamic advertising models. As our dynamics, we use a competitive extension of the Sethi [26] advertising model or a modification of the Lanchester model of combat along the lines suggested by [25,27]. The leader–follower sequence in the channel is formulated as a Stackelberg differential game, whereas the followers play a Nash differential game amongst them. Such games are quite difficult to solve. Often, only an open-loop solution is obtained, which is, in general, time-inconsistent; e.g., see [8].

This paper makes several contributions. It provides a nearly explicit (time-consistent) feedback Stackelberg equilibrium for a cooperative advertising problem with retail competition. Specifically, the equilibrium is reduced to merely solving a set of algebraic equations, which can be easily accomplished explicitly or, in some cases, numerically. We identify the levers that determine the optimal subsidy rate. Importantly, it allows us to study the effect of retail-level competition on the behavior of the manufacturer.

The rest of the paper is organized as follows. The next section reviews the relevant literature. Section 3 presents the model and the related assumptions. Section 4 presents the analysis and discusses the results. Section 5 evaluates the impact of competition on the subsidy rate. Section 6 concludes with a summary and directions for future research. Proofs of all the results in the paper are relegated to the Appendix.

2. Related literature

We first discuss static models of cooperative advertising in the literature; e.g., see [2–4,6,15,20,22]. An early paper [3] models cooperative advertising as a wholesale-price discount and finds that the manufacturer can use cooperative advertising to make higher profits. Paper [6] extends the model in [3] to study the role of cooperative advertising in franchising systems, and finds that both the franchisor and the franchisee would be better off if they jointly determine their cooperative advertising contributions than if they were to maximize their profits separately. Paper [2] explores the impact of advertising spillover and manufacturer and retailer differentiation on the subsidy rate. It shows that the subsidy rate should be higher for less differentiated retailers, more differentiated brands, and more upscale products in a product category. Paper [20] investigates minimum-advertised-price policies, where a manufacturer's payment of the advertising subsidy is contingent on the retailer's not advertising a price below the minimum-advertised-price specified by the manufacturer, and finds that the manufacturer can use a cooperative advertising subsidy along with a price floor to coordinate the channel. In an attempt to study how cooperative advertising is affected by brand name advertising, local advertising, and the subsidy rate, paper [15] analyzes two models—a traditional model with the manufacturer as the Stackelberg leader and another where the manufacturer and the retailer form a cooperative advertising partnership. The results in [15] indicate that the total channel profits and the investments in national and local advertising are higher in the partnership setting than in the traditional case.

Dutta et al. [9] conduct an empirical analysis of cooperative advertising plans offered by manufacturers to their retailers and report that the average subsidy rate over all product categories is 74.6%. More importantly, they find that the subsidy rate differs from industry to industry—it is 88.38% for consumer convenience products, 69.85% for consumer nonconvenience products, and 69.02% for industrial products.

There also exist many dynamic models of cooperative advertising in the literature (e.g., [13,14,16–19,21]). Paper [16] considers a setting in which a manufacturer and an exclusive retailer decide on advertising that has both long-term and short-term effects on sales. It finds that both channel members attain higher profits when the manufacturer supports both types of retail advertising than if it were to provide only partial advertising support. Paper [19] analyzes a goodwill model of advertising in which there is no natural channel leader. Using a dynamic incentives approach, they show that the use of cooperative advertising can generate a Pareto-optimal joint profit maximization outcome. Paper [18] analyzes a model with a manufacturer, who invests in national advertising to promote the brand's image, and a retailer, who invests in local advertising that damages the brand's image. It shows that it is optimal for the manufacturer to use cooperative advertising if the brand's image is sufficiently low, or if the harm to the brand's image from the retailer's advertising efforts is low. Paper [21] models advertising at both the manufacturer and retailer levels. It considers a retailer that sells two products – the manufacturer's product and a private label at a lower price – and show that the manufacturer can use cooperative advertising to mitigate the negative impact of the retailer's private label.

Finally, a recent paper [13] uses the Sethi [26] model in their study of a stochastic Stackelberg differential game between a manufacturer and a retailer. It obtains the optimal feedback advertising and pricing policies for the manufacturer and the retailer, and also the conditions under which the manufacturer finds it optimal to offer a cooperative advertising subsidy to the retailer.

In this paper, we extend the model in [13] to include retail competition. The next section describes our model.

3. Model

We consider a distribution channel with a single manufacturer who sells a product to a retailer (Retailer 1) who is competing with an independent retailer (Retailer 2) selling a substitute. Let $x(t)$ denote the market share of Retailer 1 at time $t \geq 0$, which depends on its own and its competitor's advertising efforts. Accordingly, the market share of the independent Retailer 2 at time t is $1 - x(t)$. The manufacturer supports Retailer 1's advertising activities by sharing a portion of the retailer's advertising expenditures. This support, termed the subsidy rate, for Retailer 1 is denoted $\theta(t)$ at time $t \geq 0$.

The advertising expenditure is quadratic in the advertising effort $u_i(t)$, $i = 1, 2$, and the manufacturer's and Retailer 1's advertising expenditure rates at time t are given by $\theta(t)u_1^2(t)$ and $(1 - \theta(t))u_1^2(t)$, respectively. The assumption of a quadratic cost function is common in the literature and implies diminishing marginal returns to advertising expenditure; e.g., see [5,7,13,25, 27].

The sequence of events in the game is as follows. The manufacturer sets the subsidy rate policy $\theta(x)$ for Retailer 1. Taking the subsidy as given, both retailers choose their advertising efforts u_1 and u_2. This effort can include store displays, circulars, and other forms of local advertising. Sales are then realized. Note that in making its decision, the retailers take each other's reaction into consideration and the manufacturer takes into account the retailers' responses.

For simplicity in exposition and to focus primarily on the effect of retail level competition, we assume that the retailers are symmetric. By this we mean that they have the same advertising response constant ρ, the same churn parameter δ, the same discount rate r, and the same profit margin m on sales. With this assumption, the market share dynamics of Retailer 1 is given by

$$\dot{x}(t) = \rho u_1(t)\sqrt{1 - x(t)} - \rho u_2(t)\sqrt{x(t)} - \delta x(t) + \delta(1 - x(t)),$$
$$x(0) = x_0 \in [0, 1], \tag{1}$$

with $1 - x(t)$ representing the market share of Retailer 2. This specification, characterized by the square-root feature introduced in [26], has the same desirable property of concave response of the steady state market share with respect to advertising as the Vidale–Wolfe [28] model. In addition, the Sethi model includes a word-of-mouth effect at the low level of market share as discussed in [26,27]. Note that the market share is non-decreasing in the retailer's own advertising effort and non-increasing in the competitor's advertising effort. Moreover, the specified concave response has been validated in empirical studies by [5,10,24].

Each retailer maximizes its profit with respect to its advertising decision in response to the subsidy policies $\theta(x)$ announced by the

manufacturer. Let $V_i(x)$ denote the value function of Retailer i at any time t when $x(t) = x$. Then $V_i(x_0)$, the optimal value of Retailer i's discounted total profit at time zero, is given by

$$V_1(x_0) = \max_{u_1(t), t \geq 0} \int_0^\infty e^{-rt}[mx(t) - (1 - \theta(x(t)))u_1^2(t)]dt, \qquad (2)$$

$$V_2(x_0) = \max_{u_2(t), t \geq 0} \int_0^\infty e^{-rt}[m(1 - x(t)) - u_2^2(t)]dt. \qquad (3)$$

Solving the Nash differential game (1)–(3) yields Retailer i's feedback advertising effort $u_i(x|\theta(x))$, $i = 1, 2$, in response to the manufacturer's announced participation rate policy $\theta(x)$.

The manufacturer anticipates the retailers' reaction functions when solving for its subsidy rate. Therefore, the manufacturer's problem is given by:

$$V(x_0) = \max_{\theta(t), t \geq 0} \int_0^\infty e^{-rt}[Mx(t) - \theta(t)(u_1(x(t)|\theta(t)))^2]dt \qquad (4)$$

subject to

$$\dot{x}(t) = \rho u_1(x(t)|\theta(t))\sqrt{1 - x(t)} - \rho u_2(x(t)|\theta(t))\sqrt{x(t)}$$
$$- \delta x(t) + \delta(1 - x(t)), \quad x(0) = x_0 \in [0, 1], \qquad (5)$$

where M is the given margin at which the manufacturer sells the product to Retailer 1.

Solution of the optimal control problems (4)–(5) requires us to obtain the optimal subsidy rates $\theta^*(t)$. This can be accomplished here by obtaining the optimal subsidy policy in feedback form, which we can, with an abuse of notation, express as $\theta^*(x)$. From this, we obtain $\theta^*(t) = \theta^*(x^*(t))$, $t \geq 0$, along the optimal path $x^*(t)$, $t \geq 0$. Furthermore, we can express Retailer i's feedback advertising effort, again with an abuse of notation, as $u_i^*(x) = u_i^*(x|\theta^*(x))$. Note that the policies $\theta^*(x)$ and $u_i^*(x)$, $i = 1, 2$, constitute a feedback Stackelberg equilibrium of the problems (2)–(5), which is time-consistent, as opposed to the open-loop Stackelberg equilibrium, which, in general, is not. Substituting these policies into the state equation in (1) yields the market share process $x^*(t)$, $t \geq 0$, and the respective decisions $\theta^*(x^*(t))$ and $u_i^*(x^*(t))$ at time $t \geq 0$.

4. Analysis and results

In this section we solve the Stackelberg differential game (2)–(5), and present the results in the following two propositions.

Proposition 1. *The feedback Stackelberg equilibrium of the game (2)–(5) is characterized as follows:*

(a) *The optimal advertising decisions of the retailers are given by*

$$u_1(x|\theta(x)) = \frac{V_1'(x)\rho\sqrt{1 - x}}{2(1 - \theta(x))},$$
$$u_2(x|\theta(x)) = -\frac{V_2'(x)\rho\sqrt{x}}{2}. \qquad (6)$$

(b) *The optimal subsidy rate of the manufacturer has the form*

$$\theta(x) = \left(\frac{2V'(x) - V_1'(x)}{2V'(x) + V_1'(x)}\right)^+. \qquad (7)$$

(c) *The value functions V_1, V_2, and V for Retailer 1, Retailer 2, and the manufacturer, respectively, satisfy the following three simultaneous differential equations:*

$$rV_1 = mx + \frac{(1 - x)\rho^2(V_1')^2}{4\left(1 - \left(\frac{2V' - V_1'}{2V' + V_1'}\right)^+\right)}$$
$$+ \frac{xV_1'V_2'\rho^2}{2} - V_1'\delta(2x - 1), \qquad (8)$$

$$rV_2 = m(1 - x) + \frac{x\rho^2(V_2')^2}{4} + \frac{(1 - x)V_1'V_2'\rho^2}{2\left(1 - \left(\frac{2V' - V_1'}{2V' + V_1'}\right)^+\right)}$$
$$- V_2'\delta(2x - 1), \qquad (9)$$

$$rV = Mx - \frac{(1 - x)\rho^2(V_1')^2\left(\frac{2V' - V_1'}{2V' + V_1'}\right)^+}{4\left(\left(\frac{2V' - V_1'}{2V' + V_1'}\right)^+ - 1\right)^2}$$
$$- \frac{(1 - x)V'V_1'\rho^2}{2\left(\left(\frac{2V' - V_1'}{2V' + V_1'}\right)^+ - 1\right)}$$
$$+ V'\left[\delta(2x - 1) - V_2'\rho^2 x/2\right]. \qquad (10)$$

Proof of Proposition 1. The Hamilton–Jacobi–Bellman (HJB) equations for Retailers 1 and 2 are given by:

$$rV_1 = \max_{u_1}\Big\{mx - (1 - \theta)u_1^2 + V_1'$$
$$\times (\rho u_1\sqrt{1 - x} - \rho u_2\sqrt{x} - \delta x + \delta(1 - x))\Big\}, \qquad (11)$$

$$rV_2 = \max_{u_2}\Big\{m(1 - x) - u_2^2 + V_2'$$
$$\times (\rho u_1\sqrt{1 - x} - \rho u_2\sqrt{x} - \delta x + \delta(1 - x))\Big\}. \qquad (12)$$

The first-order conditions for maximization yield the optimal advertising levels in Eq. (6). Substituting these solutions into (11) and (12) yields Eqs. (8)–(9).

The HJB equation for the manufacturer is

$$rV = \max_{\theta \geq 0}\Big\{Mx - \theta u_1^2(x|\theta) + V'(\rho u_1(x|\theta)\sqrt{1 - x} - \rho u_2$$
$$\times (x|\theta)\sqrt{x} - \delta x + \delta(1 - x))\Big\}. \qquad (13)$$

Using (6) in (13) and simplifying yields Eq. (14) given in Box I. The first-order condition for θ along with the requirement that $\theta \geq 0$ yields Eq. (7). Substituting this into Eq. (14) and simplifying gives Eq. (10). □

At this point, as in papers [13,26], we look for linear value functions, which work for our formulation because of the square-root feature in the dynamics (5). Specifically, we set

$$V_1 = \alpha_1 + \beta_1 x, \qquad V_2 = \alpha_2 + \beta_2(1 - x),$$
$$V_M = \alpha_M + \beta_M x \qquad (15)$$

in Eqs. (8)–(10), where the unknown parameters α_1, β_1, α_2, β_2, α_M, and β_M are constants. Then, by equating the coefficients of x on both sides of Eqs. (8)–(10), we get six simultaneous algebraic equations that can be solved to obtain the six unknown parameters, justifying the value function forms in (15). The results are provided in Proposition 2.

Proposition 2. (a) *The value functions of the players are as given in (15) with the parameters α_1, β_1, α_2, β_2, α_M, and β_M obtained from the following six algebraic equations:*

$$r\alpha_1 = \frac{\beta_1^2\rho^2}{4\left(1 - \left(\frac{2\beta_M - \beta_1}{2\beta_M + \beta_1}\right)^+\right)} + \beta_1\delta, \qquad (16)$$

$$r\beta_1 = m - \frac{\beta_1^2\rho^2}{4\left(1 - \left(\frac{2\beta_M - \beta_1}{2\beta_M + \beta_1}\right)^+\right)} - \frac{\beta_1\beta_2\rho^2}{2} - 2\beta_1\delta, \qquad (17)$$

$$rV = \max_{\theta \geq 0} \left\{ Mx - \frac{\theta \rho^2 (V_1')^2 (1-x)}{4(1-\theta)^2} - \left(\frac{(1-\theta)\rho^2 V_2' x + (2(\delta x - \delta(1-x))(1-\theta) - \rho^2 V_1'(1-x))}{2(1-\theta)} \right) V' \right\} \quad (14)$$

Box I.

$$r\alpha_2 = \frac{\beta_2^2 \rho^2}{4} + \beta_2 \delta, \quad (18)$$

$$r\beta_2 = m - \frac{\beta_2^2 \rho^2}{4} - \frac{\beta_1 \beta_2 \rho^2}{2\left(1 - \left(\frac{2\beta_M - \beta_1}{2\beta_M + \beta_1}\right)^+\right)} - 2\beta_2 \delta, \quad (19)$$

$$r\alpha_M = -\frac{\beta_1^2 \rho^2 \left(\frac{2\beta_M - \beta_1}{2\beta_M + \beta_1}\right)^+}{4\left(1 - \left(\frac{2\beta_M - \beta_1}{2\beta_M + \beta_1}\right)^+\right)^2} + \frac{\beta_1 \beta_M \rho^2}{2\left(1 - \left(\frac{2\beta_M - \beta_1}{2\beta_M + \beta_1}\right)^+\right)} + \delta \beta_M, \quad (20)$$

$$r\beta_M = M + \frac{\beta_1^2 \rho^2 \left(\frac{2\beta_M - \beta_1}{2\beta_M + \beta_1}\right)^+}{4\left(1 - \left(\frac{2\beta_M - \beta_1}{2\beta_M + \beta_1}\right)^+\right)^2} - \frac{\beta_2 \beta_M \rho^2}{2} - \frac{\beta_1 \beta_M \rho^2}{2\left(1 - \left(\frac{2\beta_M - \beta_1}{2\beta_M + \beta_1}\right)^+\right)} - 2\beta_M \delta. \quad (21)$$

(b) *The retailers' optimal advertising decisions are given by*

$$u_1^*(x) = \frac{\beta_1 \rho \sqrt{1-x}}{2\left(1 - \left(\frac{2\beta_M - \beta_1}{2\beta_M + \beta_1}\right)^+\right)}, \qquad u_2^*(x) = \frac{\beta_2 \rho \sqrt{x}}{2}. \quad (22)$$

(c) *The optimal subsidy rate of the manufacturer is a constant θ^* given by*

$$\theta^* = \left(\frac{2\beta_M - \beta_1}{2\beta_M + \beta_1} \right)^+. \quad (23)$$

Proof of Proposition 2. Using (15) in (8)–(10) and then equating the coefficients of x and the constants on both sides of the resulting equations give the algebraic equations (16)–(21). Then, since $V_1' = \beta_1$, $V_2' = \beta_2$, and $V_M' = \beta_M$, we use them in (8) and (9) to obtain (22) and (23). □

There are now two cases to consider: $\theta^* = 0$ and $\theta^* > 0$.

4.1. Case $\theta^* = 0$

Whenever this case arises, we must have the value-function coefficients satisfy the condition $\frac{2\beta_M - \beta_1}{2\beta_M + \beta_1} \leq 0$, and these coefficients must solve the following system of equations obtained from (16)–(21) by setting $\theta^* = \left(\frac{2\beta_M - \beta_1}{2\beta_M + \beta_1}\right)^+ = 0$. In Proposition 3, we also determine the precise conditions on the problem parameters for this case to arise.

With $\theta^* = 0$, it should be obvious that the two (symmetric) firms will also have identical value functions and the optimal decisions with respect to their corresponding market shares. Thus, we can also set $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$. Therefore, (16)–(21) reduce to the following system of four equations:

$$r\alpha = \frac{\beta^2 \rho^2}{4} + \beta \delta, \quad (24)$$

$$r\beta = m - \frac{3\beta^2 \rho^2}{4} - 2\beta \delta, \quad (25)$$

$$r\alpha_M = \frac{1}{2} \beta \beta_M \rho^2 + \beta_M \delta, \quad (26)$$

$$r\beta_M = M - \beta \beta_M \rho^2 - 2\beta_M \delta. \quad (27)$$

From the solution of these, we can obtain the subsidy threshold S, which when negative implies $\theta^* = 0$, and when positive implies $\theta^* > 0$. Specifically, we have the following proposition.

Proposition 3. (a) *Eqs. (24)–(27) have the solution*

$$\alpha = \frac{\beta(\beta \rho^2 + 4\delta)}{4r}, \qquad \alpha_M = \frac{M(\beta \rho^2 + 2\delta)}{2r(\beta \rho^2 + r + 2\delta)},$$
$$\beta_M = \frac{M}{\beta \rho^2 + r + 2\delta}, \quad (28)$$

where

$$\beta = \frac{\sqrt{(r + 2\delta)^2 + 3m\rho^2} - (r + 2\delta)}{3\rho^2 / 2}. \quad (29)$$

(b) *The subsidy threshold S is given by*

$$S = M - \frac{m}{2} - \frac{\left[\sqrt{(r + 2\delta)^2 + 3m\rho^2} - (r + 2\delta)\right]^2}{18\rho^2}. \quad (30)$$

The manufacturer chooses $\theta^* > 0$ when $S > 0$ and $\theta^* = 0$ when $S \leq 0$.

Proof of Proposition 3. Solving the quadratic equation (25) for β gives the solution as in (29). Then, we use (24), (26), and (27) to obtain the other coefficients in terms of β as given in (29).

Since $\beta_M > 0$ from Eq. (21), the subsidy threshold is obtained as $S = 2\beta_M - \beta$. We have

$$\frac{2M}{\beta \rho^2 + (r + 2\delta)} - \beta = \frac{2M - \beta^2 \rho^2 - \beta(r + \delta + \delta)}{\beta \rho^2 + (r + 2\delta)}. \quad (31)$$

From Eq. (25), we have $\beta^2 \rho^2 + 4\beta\delta = (4m - 4r\beta - \beta^2 \rho^2)/2$, which we can use in the numerator of (31) to obtain (30). □

While (30) gives the precise value of S and while it is suitable for empirical testing, it is not easy to interpret. To gain further insight, let us approximate S when r and δ are small compared to m, M, and ρ. This is done by setting $r = \delta = 0$ in (30):

$$S \approx M - 2m/3. \quad (32)$$

This means that roughly $\theta^* > 0$ if $M > 2m/3$. In other words, the manufacturer provides advertising support to Retailer 1 if $M > 2m/3$. Stated differently, if m increases by ϵ, then M must increase by $2\epsilon/3$ to maintain $S = 0$.

4.2. Case $\theta^* > 0$

When the expression in (30) results in $S > 0$, we know that $\theta^* > 0$. Then, by substituting $\left(\frac{2\beta_M - \beta_1}{2\beta_M + \beta_1}\right)^+ = \frac{2\beta_M - \beta_1}{2\beta_M + \beta_1}$ into
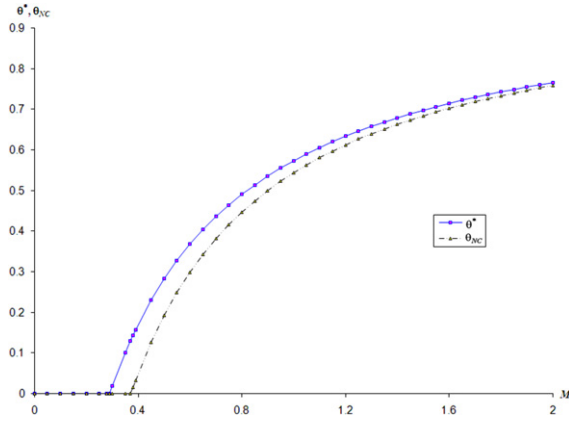
Fig. 1. Effect of competition on the optimal subsidy rate.

Fig. 2. Optimal advertising spending and market shares over time.

Eqs. (16)–(21), we have the following system of equations to solve for the value-function coefficients:

$$r\alpha_1 = \frac{1}{8}\beta_1\rho^2(2\beta_M + \beta_1) + \beta_1\delta, \tag{33}$$

$$r\beta_1 = m - \frac{1}{8}\beta_1\rho^2(2\beta_M + \beta_1) - \frac{1}{2}\beta_1\beta_2\rho^2 - 2\beta_1\delta, \tag{34}$$

$$r\alpha_2 = \frac{1}{4}\beta_2^2\rho^2 + \beta_2\delta, \tag{35}$$

$$r\beta_2 = m - \frac{1}{4}\beta_2\rho^2(2\beta_M + \beta_1) - \frac{1}{4}\beta_2^2\rho^2 - 2\beta_2\delta, \tag{36}$$

$$r\alpha_M = \frac{1}{16}\rho^2(2\beta_M + \beta_1)^2 + \beta_M\delta, \tag{37}$$

$$r\beta_M = M - \frac{1}{16}\rho^2(2\beta_M + \beta_1)^2 - \frac{1}{2}\beta_2\beta_M\rho^2 - 2\beta_M\delta. \tag{38}$$

The solution of Eqs. (33)–(38) can be obtained easily via numerical analysis.

5. Effect of competition

In order to study the effect of competition, we need to compare our results to those obtained in [13] in the case of a one-manufacturer, one-retailer channel. When parameters r and δ are small, [13] shows that the manufacturer supports the retailer when $M \geq m$. In our case, as specified in (32), the manufacturer will support Retailer 1 when $M \geq 2m/3$. Thus, we see that when there is a competing retailer, there is a greater range of retailer margin values for which the manufacturer supports the retailer. In Fig. 1, we provide the optimal subsidy sates θ^* in our case and θ_{NC} in the case of one retailer (i.e., in the no-competition case), when $m = 0.5, r = 0.03, \rho = 0.5, \delta = 0.1$. We see that the presence of a competing retailer induces the manufacturer to provide a higher level of cooperative advertising support to its retailer than if that retailer were a monopolist.

In addition to the higher subsidy rate in the competitive case, we can also note the following from Fig. 1. First, the threshold value of M at which the manufacturer supports its retailer in the presence of competition is lower than that without competition. In particular, the manufacturer sets $\theta > 0$ when $M \approx 0.29$, and $\theta_{NC} > 0$ when $M \approx 0.38$. This finding is in line with Proposition 3. Second, while the subsidy rate under competition is higher than that in the absence of competition, the difference in subsidy rates declines as M increases.

We have repeated this analysis by fixing M and varying ρ. Once again, we see that the manufacturer will offer greater subsidy to
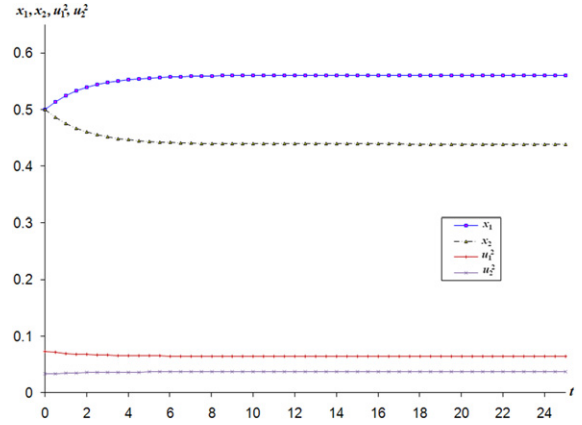
the supported retailer under competition than under no retail-level competition.

Fig. 2 provides a plot of the two retailer's market shares and their optimal advertising spending over time, starting from an equal market split. It uses $m = 0.5, r = 0.03, \rho = 0.5, \delta = 0.1$ as in Fig. 1, and $M = 0.5$. The market shares of the two retailers sum to one, and from the initial value, there is convergence to a steady state. The supported retailer advertises more on account of the subsidy received from the manufacturer, which results in an increased market share. Moreover, advertising being proportional to the competitor's market share, the supported retailer's advertising decreases as its market share increases towards the steady state. Note that advertising is required to alter and maintain the market shares, including at steady state.

6. Conclusions

In this paper, we consider a manufacturer who sells a product to its retailer competing with another independent and otherwise symmetric retailer. To preserve tractability, we did not model the manufacturer's wholesale price decision in this paper, as was done in [13]. Future research could extend the model to also include the manufacturer's wholesale price. The interaction of other variables, notably price, deserves further attention. If prices are accommodated as in [1], then the result would depend on the specification of the competitive demand function. The margins, which are exogenous parameters here, would now be endogenous and will depend on the nature of the demand function. Clearly, pricing will play a role in the co-op subsidy decision, and this interaction in a retail competition environment deserves further study. We expect that the manufacturer's support for its retailer will likely still be higher under competition than in its absence.

It would also be interesting to explore decisions in a retail oligopoly, along the lines of [12,24]. Analysis of our model shows that duopoly retail competition results in an increase in cooperative advertising support. While this result appears to be intuitive, it would be useful to confirm if it still holds true for competition in general, i.e., possibly extending to competing channel structures.

Finally, this paper opens up a fruitful avenue for future empirical research. Since we have provided a game framework for obtaining the subsidy threshold and the optimal subsidy rate that depend on various firm- and industry-level parameters, it makes it possible to empirically examine whether our results can explain the subsidy rates in different industries. This would of course require estimation of the firm- and industry-level parameters, possibly employing the techniques used in [24]. Provided an appropriate data set can be found or collected, an empirical study to validate our results would undoubtedly deepen our understanding of the cooperative advertising practices in different industries.

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